The Logic of Compound Statements

Conditional Statements; Valid and Invalid Arguments

Lecture Notes #3

Peter Blomgren
Department of Mathematics and Statistics
San Diego State University
San Diego, CA 92182-7720
blomgren@terminus.SDSU.EDU
http://terminus.SDSU.EDU

$Id: lecture.tex,v 1.16 2006/09/07 18:24:34 blomgren Exp $

Previously:— Logical Form and Equivalence 1 of 2

Statements: Sentences that are either **TRUE** or **FALSE**, but not both.

Logical symbols: $\sim$ not

$\wedge$ and

$\vee$ or

Statement form: An expression made up of statement variables and symbols that ***becomes*** a statement when actual statements are substituted for the statement variables.

Truth table: A table showing all possible truth-value combinations of the **statement variables** (p, q, r, $\ldots$), as well as the corresponding truth values for a simple, or compound, statement of interest. (In the case of a compound statement, we also tend to include columns for intermediate statements.)

Logical equivalence: Two logical expressions with the **same** truth values (columns in a truth table), are said to be ***logically equivalent*** (*i.e.* two different ways of expressing the "same thing.")

Tautology: A logical expression that is always **`true`** (for all "input" logical variables.) *E.g.* $p \vee (\sim p)$.

Contradiction: A logical expression that is always **`false`** (for all "input" logical variables.) *E.g.* $p \wedge (\sim p)$.

A logical inference or deduction is made ***from*** a hypothesis ***to*** a conclusion.

Let p and q be statements. A sentence of the form "if p then q" is denoted by

$$p \rightarrow q$$

p is the ***hypothesis***, and q the ***conclusion***.

$\rightarrow$ is a logical connective, and like $\wedge$, $\sim$ and $\vee$ it can be used to join statements to create new statements.

To define $p \rightarrow q$ as a statement, we must specify the truth values for $p \rightarrow q$ just as we did for $p \wedge q$ (and friends).

If-Then ($\rightarrow$) Truth Table

The formal definition of truth values for $\rightarrow$ is based on its everyday intuitive meaning.

The promise "***If*** you show up for class on Tuesday, ***then*** you will get an A in this class" is **false** only if you **do** show up for class on Tuesday, and **do not** get an A in this class. In all other cases it is true (the promise is not broken.)

Hence the truth table looks like:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example: Truth Table for $p \vee (\sim q) \rightarrow (\sim p)$

Recall:

***Definition:* Conditional** —
If p and q are statement variables, the **conditional** of q by p is "if p then q" or "p implies q" and is denoted $p \rightarrow q$. It is false when p is true and q is false; otherwise it is true.

p	q	$\sim p$	$\sim q$	$p \vee (\sim q)$	$p \vee (\sim q) \rightarrow (\sim p)$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

“Vacuously True” / “True By Default”

A conditional statement ($p \rightarrow q$) that is **true** by virtue of the fact that the hypothesis (p) is **false** is often called **vacuously true** or **true by default**.

The statement *“If you show up for work on Tuesday morning, then you will get the job”* is vacuously **true** if you do not show up for work on Tuesday morning. (In this case there is ***no*** promise, hence it cannot be broken.)

Logical Equivalences Involving $\to$

Example: Showing that $(p \lor q) \to r \equiv (p \to r) \land (q \to r)$

p	q	r	$(p \lor q)$	$p \to r$	$q \to r$	$(p \lor q) \to r$	$(p \to r) \land (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Since the last two columns match, we have shown that
$(p \lor q) \to r \equiv (p \to r) \land (q \to r)$.

Negation of a Conditional Statement

The negation of "if p then q" is logically equivalent to "p and not q"

Proof:

p	q	$p \to q$	$\sim q$	$\sim (p \to q)$	$p \wedge \sim q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	F	F

Example: (Note that we use $\sim (\sim q) \equiv q$)

$\sim$ "If my car is in the shop, then I cannot get to class."

$\equiv$ "My car is in the repair shop, and I can get to class."

The Contrapositive of a Conditional Statement

Definition: ***Contrapositive —***

The **contrapositive** of a conditional statement of the form "if p then q" is,

"If $(\sim q)$ then $(\sim p)$"

Symbolically, the contrapositive of $(p \to q)$ is $((\sim q) \to (\sim p))$.

You will be asked (see homework) to show that ***A conditional statement is logically equivalent to its contrapositive***, *i.e.*

$$(p \to q) \equiv ((\sim q) \to (\sim p))$$

Examples: Writing the Contrapositive

(#1) The contrapositive of:

"If Howard can swim across the lake, then Howard can swim to the island."

is

"If Howard cannot swim to the island, then Howard cannot swim across the lake."

(#2) The contrapositive of:

"If today is Easter, then tomorrow is Monday."

is

"If tomorrow is not Monday, then today is not Easter."

The Contrapositive is an Important Tool

We will see the contrapositive form later on in this class:

The logical equivalence of a conditional statement and its contrapositive is the basis for one of the laws of deduction (***modus tollens***), and for the ***contrapositive method of proof.***

The Inverse and Converse of a Conditional Statement

Definition: ***Converse and Inverse* —**

Suppose a conditional statement of the form "if p then q is given.

(#1) The **converse** is "if q then p"

(#2) The **inverse** is "if $(\sim p)$ then $(\sim q)$"

Symbolically,

$$\text{The converse of } (p \to q) \text{ is } (q \to p)$$

$$\text{The inverse of } (p \to q) \text{ is } ((\sim p) \to (\sim q))$$

Note: The inverse and converse are ***not*** logically equivalent to the statement; they are, however, logically equivalent to each other, since the inverse is the contrapositive of the converse.

Midterm alert!

“Only if”

To say “p only if q” means that p can take place only if q takes place also. That is, if q does not take place, then p cannot take place.

By the logical equivalence of the contrapositive, we can also say that if p occurs, then q must also occur.

Definition: ***Only If*** —

If p and q are statements,

p **only if** q means “if not q then not p”

or equivalently,

“if p then q.”

"If, and only if" — The Bi-conditional

Definition: ***If, and Only If*** —

Given the statement variables p and q, the **bi-conditional of** p **and** q is "p if, and only if, q" and is denoted $(p \leftrightarrow q)$. It is true if both p and q have the same truth values, and is false if p and q have opposite truth values. The words ***if and only if*** are sometimes abbreviated ***iff***.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Order of Operations

In order of operations $\leftrightarrow$ is co-equal with $\rightarrow$, and we have the following precedence for our five logical connectives

highest	1	$\sim$
$\downarrow$	2	$\wedge$, $\vee$
lowest	3	$\rightarrow$, $\leftrightarrow$

Order of operations

"if", "only if" and "if, and only if"

According to the definitions of "if" and "only if", saying "p if, and only if q" should mean the same as saying "p if q" and "p only if q." That is indeed the case... again we look at the truth table.

p	q	p only if q $(p \rightarrow q)$	p if q $(q \rightarrow p)$	p iff q $(p \leftrightarrow q)$	(p only if q) and (p if q) $(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Since the last two columns are equal, the statement forms are equivalent, *i.e.* $(\mathbf{p} \leftrightarrow \mathbf{q}) \equiv (\mathbf{p} \rightarrow \mathbf{q}) \wedge (\mathbf{q} \rightarrow \mathbf{p})$.

Necessary and Sufficient Conditions

The phrases ***necessary condition*** and ***sufficient condition***, as used in formal English correspond exactly to their definitions in logic:

Definition: ***Sufficient and Necessary Conditions —***
If r and s are statements:

r is a **sufficient condition** for s means "if r then s"

r is a **necessary condition** for s means "if not r then not s"

Note that due to the equivalence between a statement and its contra-positive:

r is a **necessary condition** for s also means "if s then r"

Epp-1.2.28: "Do you mean that you think you can find out the answer to it" said the March Hare.

"Exactly so," said Alice.

"Then you should say what you mean," the March Hare went on.

"I do," Alice hastily replied; "at least — at least I mean what I say — that's the same thing you know."

"Not the same thing a bit!" said the Hatter. "Why, you might just as well say that "I see what I eat" is the same thing as "I eat what I see!"

—from "A Mad Tea Party" in *Alice in Wonderland,* by Lewis Carroll.

That Hatter is right. "I say what I mean" is not the same thing as "I mean what I say." Rewrite in if—then form, and explain the difference.

The if–then form of "I say what I mean" is

"If I mean something, then I say it."

$$(\text{mean}) \longrightarrow (\text{say})$$

The if–then form of "I mean what I say" is

"If I say something, then I mean it."

$$(\text{say}) \longrightarrow (\text{mean})$$

The two statements are the ***converse*** of each other, and are ***not logically equivalent***.

Corresponds to Epp-v2.0-1.2.24

Homework #1 — Due 9/15/2006, 12:00pm, GMCS-587

Epp-1.2: 13, 24, 25, 26, 27

Epp-1.1: 3, 14, 16, 21, 25, 29, 31, 37, 41

Extra Brain-Twister (for fun): ***Epp-1.1.54***

Arguments — Introduction

We are now going to use our new tools / language — logic statements, connectives, conditionals... to generate ***arguments***.

In mathematics / logic an argument is not a dispute, rather...

> ***Definition:*** ***Argument*** —
>
> An **argument** is a sequence of statements. All statements but the final one are called **premises** (or **assumptions** or **hypotheses**). The final statement is called the **conclusion**. The symbol "$\therefore$", read "therefore," is normally placed just before the conclusion.

We will be concerned with determining whether an argument is ***valid***, that is, to determine whether the conclusion follows necessarily from the preceding statements.

Abstracting the Content from the Arguments

We have already seen (Lecture Notes #2) that we can separate the content from the argument, recall:

Statement A:

If Jane is a math major or Jane is a CS major,
then Jane will take Math 245.
Jane is a CS major.
Therefore, Jane will take Math 245.

Abstract logical form	**With our new symbol**
If **p** or **q**, then **r**.	If **p** or **q**, then **r**.
q.	**q**.
Therefore, **r**	$\therefore$ **r**

Valid Arguments

When we consider the abstract form of an argument, *e.g.*

If **p** or **q**, then **r**.
q.
∴ **r**

we think of p, q, and r as variables for which statements may be substituted.

Definition: ***Valid Argument Form*** —

To say that an ***argument form*** is **valid** means that no matter what particular statements are substituted for the statement variables in its premises, if the resulting premises are *all true*, then the conclusion is also true.

To say that ***an argument*** is valid means that its form is valid.

Valid Arguments

The truth of the conclusion of a valid argument follows ***necessarily*** or ***inescapably*** or ***by logic alone*** from the truth of its premises.

It is impossible to have a valid argument with true premises and a false conclusion.

When an argument is valid and its premises are true, the truth of the conclusion is said to be ***inferred*** or ***deduced*** from the truth of the premises.

Testing for Validity

To test an argument form for validity:

(1) Identify the premises and conclusion of the argument.

(2) Construct a truth table showing the truth values of all the premises and the conclusion.

(3) Find the ***critical rows*** in which all the premises are true.

(4) In each critical row, determine whether the conclusion of the argument is also true.

- **(a)** If in each critical row the conclusion is also true, then the argument form is valid.

- **(b)** If there is at least one critical row in which the conclusion is false, the argument form is invalid.

Show that the following argument form is valid:

$$p \vee (q \vee r)$$

$$\sim r$$

$$\therefore \quad (p \vee q)$$

variables				premises		conclusion
p	q	r	$(q \vee r)$	$p \vee (q \vee r)$	$\sim r$	$(p \vee q)$
T	T	T	T	T	F	–
T	**T**	**F**	**T**	**T**	**T**	**T**
T	F	T	T	T	F	–
T	**F**	**F**	**F**	**T**	**T**	**T**
F	T	T	T	T	F	–
F	**T**	**F**	**T**	**T**	**T**	**T**
F	F	T	T	T	F	–
F	F	F	F	F	T	–

Show that the following argument form is invalid:

$$p \to q \vee \sim r$$

$$q \to p \wedge r$$

$$\therefore \quad p \to r$$

variables						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$	$q \to p \wedge r$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	–
T	F	T	F	F	T	F	T	–
T	F	F	T	T	F	**T**	**T**	**F**
F	T	T	F	T	F	T	F	–
F	T	F	T	T	F	T	F	–
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

Modus Ponens — The Method of Affirming

If we have an argument of the form:

If **p**, then **q**.
p.
$\therefore$ **q**

The fact that this argument forms is valid is called **modus ponens** (from Latin).

		premises		conclusion
p	q	$p \rightarrow q$	p	q
T	T	T	T	T
T	F	F	T	–
F	T	T	F	–
F	F	T	F	–

Modus Tollens — The Method of Denying

If we have an argument of the form:

If **p**, then **q**. $\sim$ **q**. $\therefore \sim$ **p**	$\equiv$ contrapositive	If $\sim$ **q**, then $\sim$ **p**. $\sim$ **q**. $\therefore \sim$ **p**

The fact that this argument forms is valid is called **modus tollens** (from Latin).

		premises		conclusion
p	q	$p \rightarrow q$	$\sim q$	$\sim p$
T	T	T	F	–
T	F	F	T	–
F	T	T	F	–
F	F	T	T	T

Disjunctive addition is used for making generalizations:

p
$\therefore \quad (p \vee q)$

q
$\therefore \quad (p \vee q)$

		premises	conclusion
p	q	p	$p \vee q$
T	T	T	T
T	F	T	T
F	T	F	–
F	F	F	–

		premises	conclusion
p	q	q	$p \vee q$
T	T	T	T
T	F	F	–
F	T	T	T
F	F	F	–

Example: Students (p) and [LOGICAL OR] Seniors (q) get a discount at store X. You are a student (p), therefore ($(p \vee q)$) you get a discount.

Conjunctive Simplification — Specialization

Conjunctive simplification is used for particularizing:

	$p \wedge q$
$\therefore$	p

	$p \wedge q$
$\therefore$	q

		premises	conclusion
p	q	$p \wedge q$	p
T	T	T	T
T	F	F	–
F	T	F	–
F	F	F	–

		premises	conclusion
p	q	$p \wedge q$	q
T	T	T	T
T	F	F	–
F	T	F	–
F	F	F	–

Example: You are tired of logic and Peter. Therefore (in particular) you are tired of logic.

Disjunctive Syllogisms are used to rule out possibilities:

	$(p \lor q)$
	$\sim q$
$\therefore$	p

	$(p \lor q)$
	$\sim p$
$\therefore$	q

		premises		conclusion
p	q	$p \lor q$	$\sim q$	p
T	T	T	F	–
T	F	T	T	T
F	T	T	F	–
F	F	F	T	–

		premises		conclusion
p	q	$p \lor q$	$\sim p$	q
T	T	T	F	–
T	F	T	F	–
F	T	T	T	T
F	F	F	T	–

Example: You are tired of logic or surfing. You are not tired of surfing. Therefore you are tired of logic.

Hypothetical Syllogisms are used to build chains of implication:

$$\begin{array}{rl} & p \rightarrow q \\ & q \rightarrow r \\ \therefore & p \rightarrow r \end{array}$$

			premises		conclusion
p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$
T	T	T	T	T	T
T	T	F	T	F	–
T	F	T	F	–	–
T	F	F	F	–	–
F	T	T	T	T	T
F	T	F	T	F	–
F	F	T	T	T	T
F	F	F	T	T	T

Hypothetical Syllogism — Example

"If it is sunny, the sky is blue"

"If the sky is blue, we'll go surfing"

Therefore, "If it is sunny, we'll go surfing"

$$
\begin{array}{ll}
 & (\text{sunny}) \longrightarrow (\text{sky blue}) \\
 & (\text{sky blue}) \longrightarrow (\text{surfing}) \\
\therefore & (\text{sunny}) \longrightarrow (\text{surfing})
\end{array}
$$

The following statements are true:

a. If $\underbrace{\text{my glasses are on the kitchen table}}_{p}$, then $\underbrace{\text{I saw my glasses at breakfast}}_{q}$. $(p \to q)$

b. $\underbrace{\text{I was reading the newspaper in the living room}}_{r}$, or $\underbrace{\text{I was reading the newspaper in the kitchen}}_{s}$. $(r \vee s)$

c. If r then $\underbrace{\text{my glasses are on the coffee table}}_{t}$. $(r \to t)$

d. I did not see my glasses at breakfast. $(\sim q)$

e. If $\underbrace{\text{I was reading my book in bed}}_{u}$, then $\underbrace{\text{my glasses are on the bed table}}_{v}$. $(u \to v)$

f. If s, then p. $(s \to p)$

We have the following:

a. $(p \rightarrow q)$ **b.** $(r \vee s)$ **c.** $(r \rightarrow t)$

d. $(\sim q)$ **e.** $(u \rightarrow v)$ **f.** $(s \rightarrow p)$

We make the following deductions:

1. By **a** and **d**, we deduce $(\sim p)$, by modus tollens.

2. By **f** and **1**, we deduce $(\sim s)$, by modus tollens.

3. By **b** and **2**, we deduce (r), by disjunctive syllogism.

4. By **c** and **3**, we deduce (t), by modus ponens.

Hence, the glasses are on the coffee table.

Fallacies — Broken Logic

A **fallacy** is an error in reasoning resulting in an invalid statement.

Three common mistakes:

(1) Using vague or ambiguous premises.
(2) Assuming what is to be proved.
(3) Jumping to conclusions without adequate grounds.

In the next few slides we'll explore two other fallacies:

(4) Converse Error
(5) Inverse Error

Which give rise to arguments which resemble modus ponens and modus tollens, but are invalid.

Checking for Fallacies

There are two ways...

(1) Construct the truth table, and demonstrate that there is at least one critical row in which the premises are true, but the conclusion false.

(2) Find an argument of the same form (logical equivalence) with true premises and a false conclusion. (Counter-example)

Converse Error

> If Peter is a cheater, then Peter will sit in the back row.
>
> Peter sits in the back row.
>
> Therefore Peter is a cheater.

It is quite possible that Peter is not a cheater, but is sitting in the back row!

You will be asked (homework) to construct the truth table, showing that this type of argument is invalid.

Inverse Error

If Peter is a cheater, then Peter will sit in the back row.

Peter is not a cheater.

Therefore Peter does not sit in the back row.

It is quite possible that Peter is not a cheater, even though he is sitting in the back row!

You will be asked (homework) to construct the truth table, showing that this type of argument is invalid.

Homework #1 — Due 9/15/2006, 12:00pm, GMCS-587

Epp-1.3: 13, 21, 39

Epp-1.3: Read examples 1.3.15, 1.3.16

Epp-1.2: 13, 24, 25, 26, 27

Epp-1.1: 3, 14, 16, 21, 25, 29, 31, 37, 41

Extra Brain-Twister (for fun): ***Epp-1.1.54***

A lot of the theory of symbolic logic we have seen so far was developed by Augustus ***De Morgan*** (1806–1871) and George ***Boole*** (1815–1864), in the 19th century.

One of the "cleanest" application of logic "in the wild" is to construction of digital logic circuits.

In essence, a processor chip is nothing but a huge collection of AND-, OR-, and NOT-switches.

Claude ***Shannon*** (1916–2001) made the connection between switched systems and logic, and used formal logic to solve circuit design problems. His master's thesis *A Symbolic Analysis of Relay and Switching Circuits* was published in 1938.

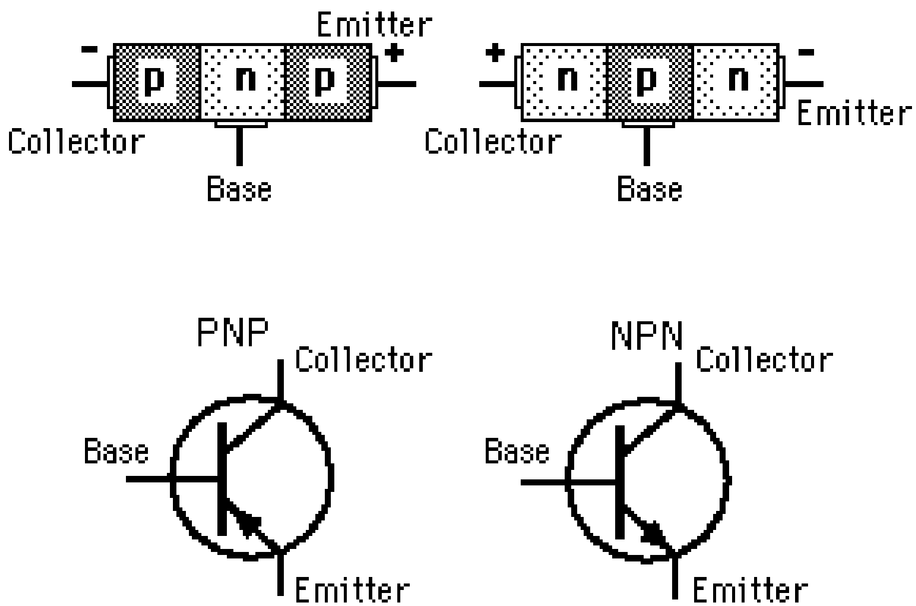
Claude Shannon's doctoral thesis was on ***theoretical genetics.*** His paper ***A Mathematical Theory of Communication*** (1948) founded the subject of information theory. — The idea that one could transmit pictures, words, sounds etc. by sending a stream of 1's and 0's down a wire, was fundamentally new.

In 1956, William Bradford Shockley (1910–1989), John Bardeen (1908–1991), and Walter Houser Brattain (1902–1987) received the Nobel Prize in Physics ***"for their researches on semiconductors and their discovery of the transistor effect."***

— The transistor is the small semiconductor device which makes modern computers possible.

The Transistor

We'll take a quick look at how to build logic circuits, using the transistor as a building block... First, let's look at the transistor:

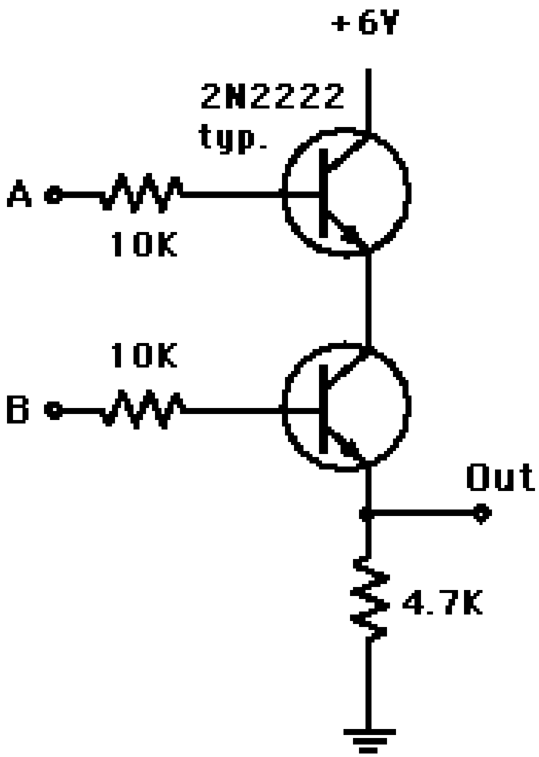


"A bipolar junction transistor consists of three regions of doped semiconductors. A small current in the center or base region can be used to control a larger current flowing between the end regions (emitter and collector). The device can be characterized as a current amplifier, having many applications for amplification and switching."

Note: Figures and text "borrowed" from

`http://hyperphysics.phy-astr.gsu.edu/`

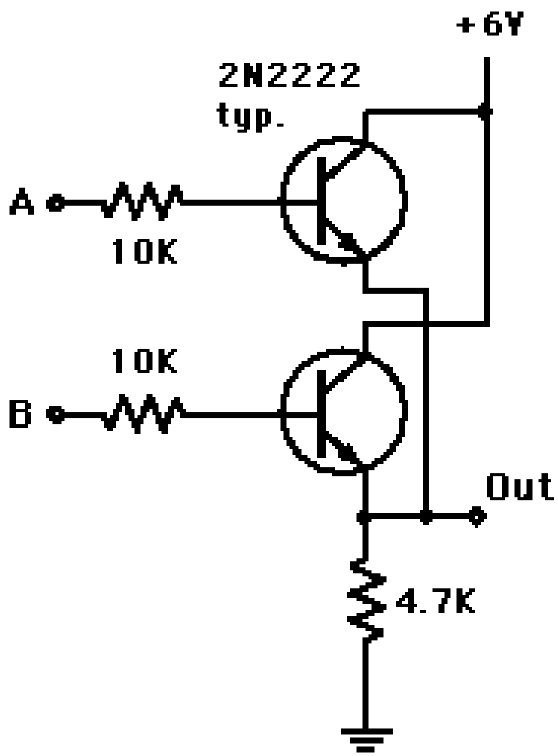
The Transistor AND Gate $A \wedge B$



"The use of transistors for the construction of logic gates depends upon their utility as fast switches. When the base-emitter diode is turned on enough to be driven into saturation, the collector voltage with respect to ground may be less than a volt and can be used as a logic 0 in the TTL logic family."

Here, if we connect a **`true`** value ("1", or +6V) to both A and B, then both transistors open, and the ***out*** value is "1." Otherwise there is no connection to +6V from the **`out`**, hence the value is **`false`** ("0", or 0V).

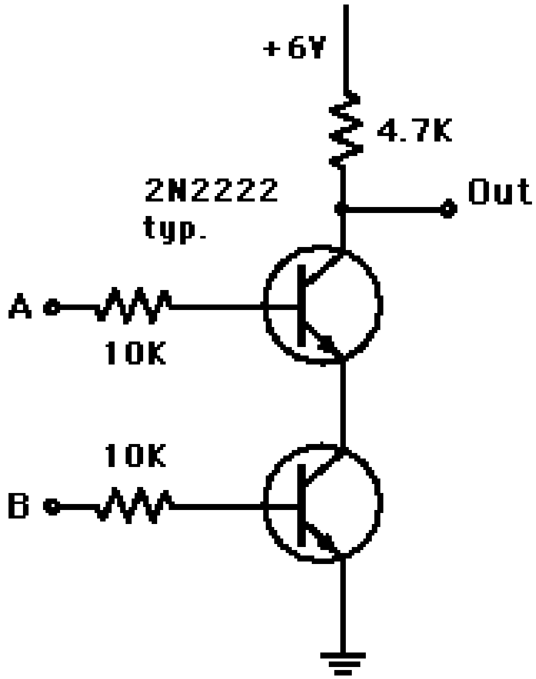
The Transistor OR Gate $A \vee B$



"The use of transistors for the construction of logic gates depends upon their utility as fast switches. When the base-emitter diode is turned on enough to be driven into saturation, the collector voltage with respect to ground may be less than a volt and can be used as a logic 0 in the TTL logic family."

Here, if we connect a **`true`** value ("1", or +6V) to at least one of A and B, then there is a path from ***out*** to +6V, and the output value if **`true`**. Otherwise there is no connection to +6V from the **`out`**, hence the value is **`false`** ("0", or 0V).

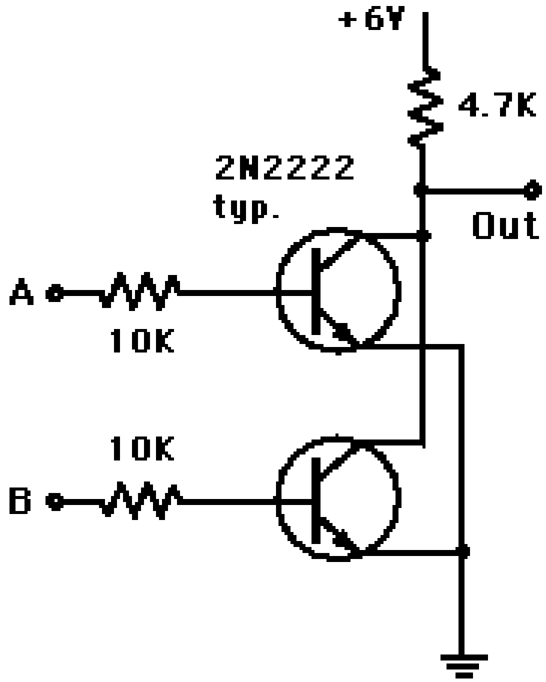
The Transistor NAND Gate $\sim (A \wedge B)$



"The use of transistors for the construction of logic gates depends upon their utility as fast switches. When the base-emitter diode is turned on enough to be driven into saturation, the collector voltage with respect to ground may be less than a volt and can be used as a logic 0 in the TTL logic family."

Here, if we connect a **`true`** value ("1", or +6V) to both A and B, then both transistors open, and the **`out`** value is "0." Otherwise there is no connection to 0V from the **`out`**, hence the value is **`true`** ("1", or +6V).

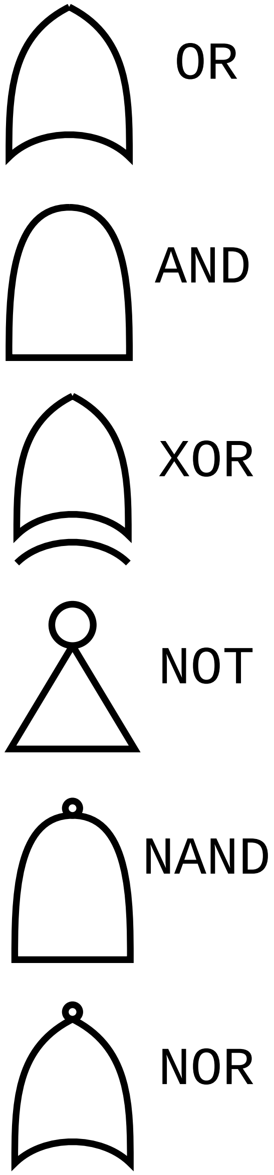
The Transistor NOR Gate $\sim (A \lor B)$



"The use of transistors for the construction of logic gates depends upon their utility as fast switches. When the base-emitter diode is turned on enough to be driven into saturation, the collector voltage with respect to ground may be less than a volt and can be used as a logic 0 in the TTL logic family."

Here, if we connect a **`true`** value ("1", or +6V) to at least one of A and B, then there is a path from ***to*** to 0V, and the **`out`** value if **`false`**. Otherwise there is no connection to 0V from the **`out`**, hence the value is **`true`** ("1", or +6V).

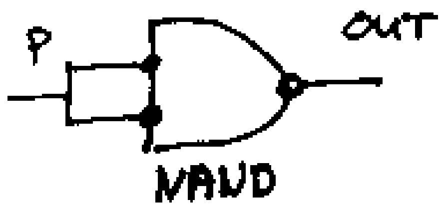
Standard Circuit Symbols



To the left we see the standard circuit symbols for common logical connectives.

Note that we can build the missing ones (XOR and NOT) from the ones we already have (OR, AND, NAND, NOR).

Building a NOT circuit...

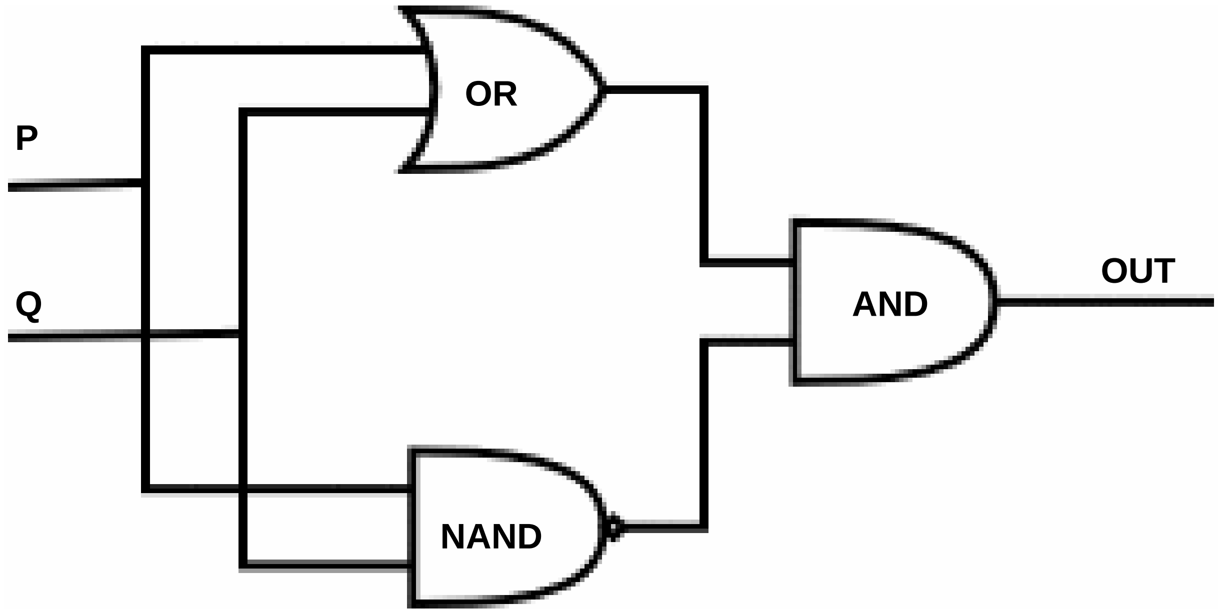


P	OUT $\equiv \sim P$
0	1
1	0

By connecting the input (P) to both in-ports on the NAND-gate we get an inverter (NOT-gate).

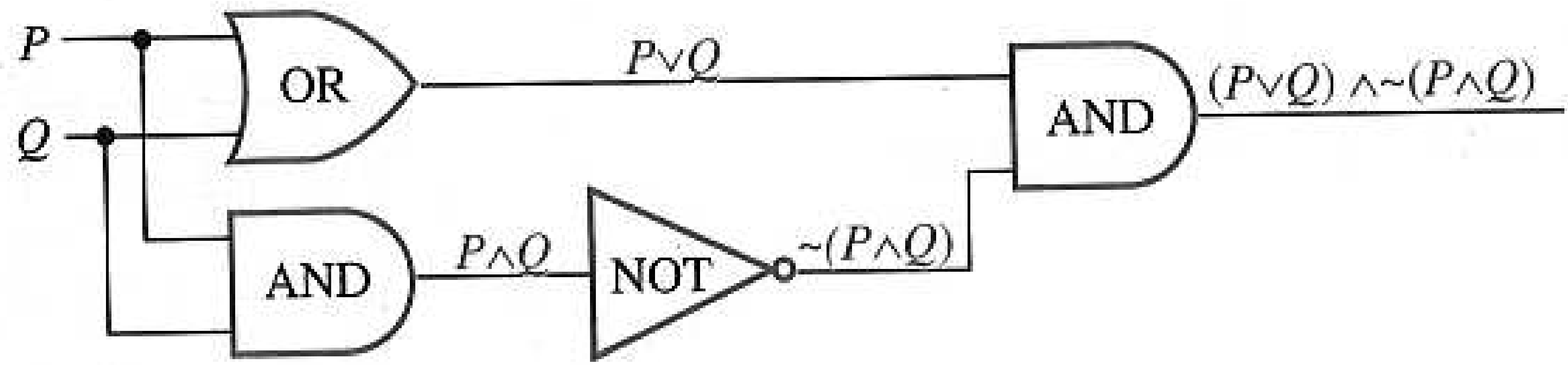
Building an XOR circuit...

(P OR Q) AND NOT (P AND Q)



p	q	$p \vee q$	$p \wedge q$	$\sim(p \wedge q)$	$(p \vee q) \wedge \sim(p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Finding the Logic (Boolean) Expression for a Circuit

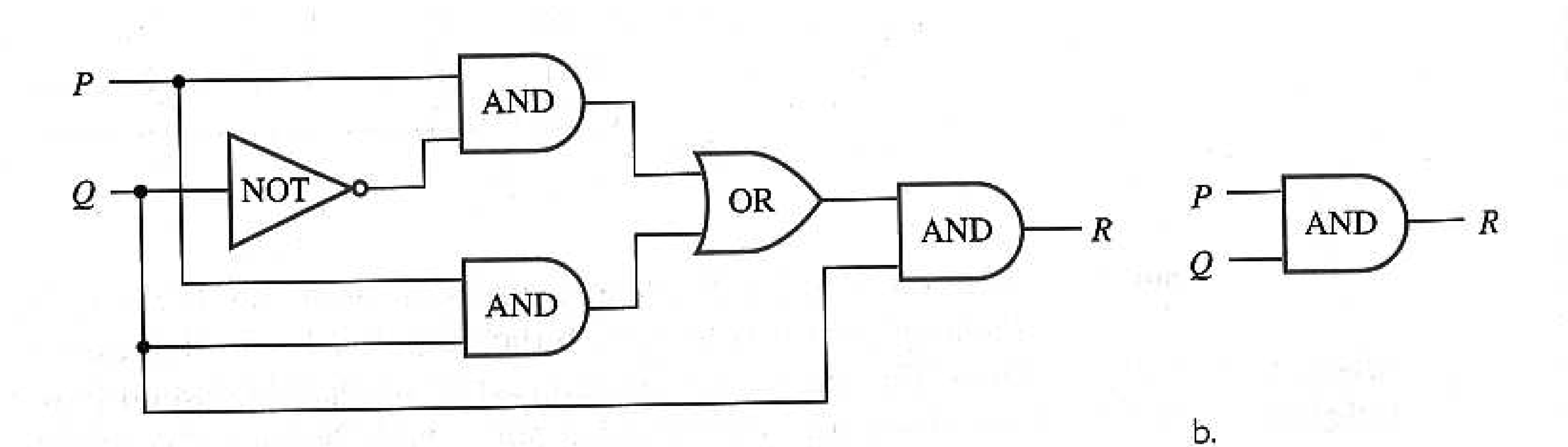


In order the find the expression for a circuit, for each gate simply apply the appropriate operation to the inputs.

The Input/Output Table for a Circuit

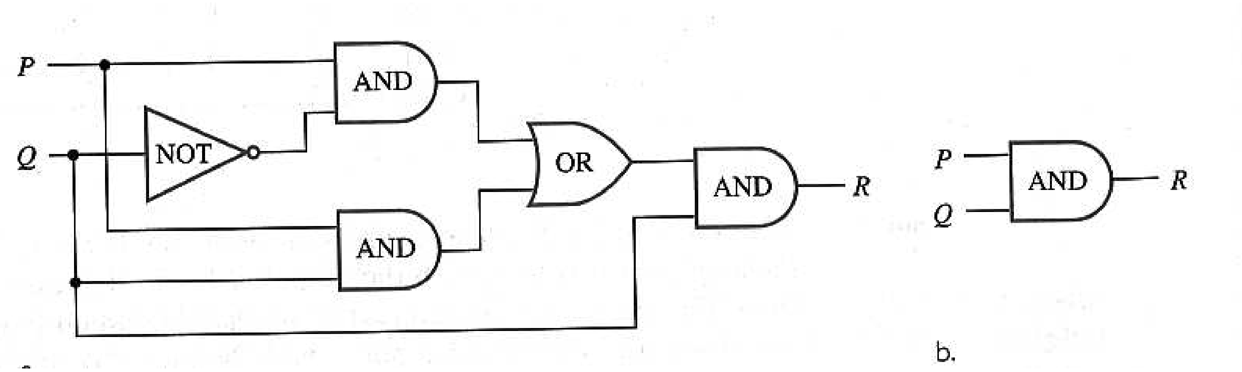
The Input/Output table for a circuit is a table (much like the truth table) which shows the output value of the circuit, for all possible combinations of inputs.

Two circuits are **equivalent** if, and only if, their input/output tables are identical.



Example of two equivalent circuits.

Showing that Two Circuits are Equivalent



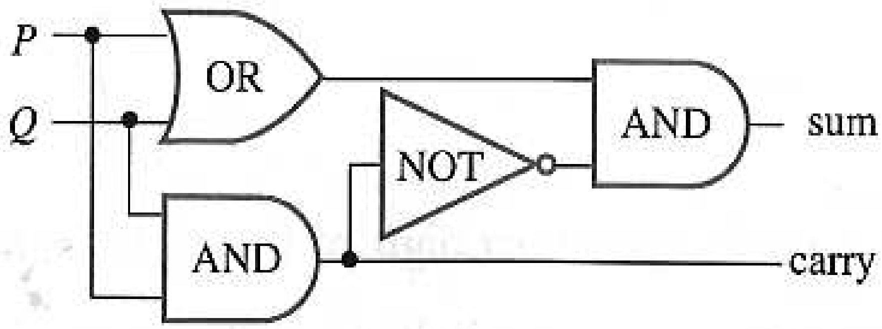
Example of two equivalent circuits.

We can ***either*** construct the input/output tables for the circuits and check that the tables are identical; ***or*** we can use our knowledge of symbolic logic.

For the circuit above:

$((P \wedge \sim Q) \vee (P \wedge Q)) \wedge Q \equiv$	distributive law
$(P \wedge (\sim Q \vee Q)) \wedge Q \equiv$	negation law
$(P \wedge t) \wedge Q \equiv P \wedge Q$	identity law

Adding Bits with Circuits



P	Q	Carry	Sum
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	0

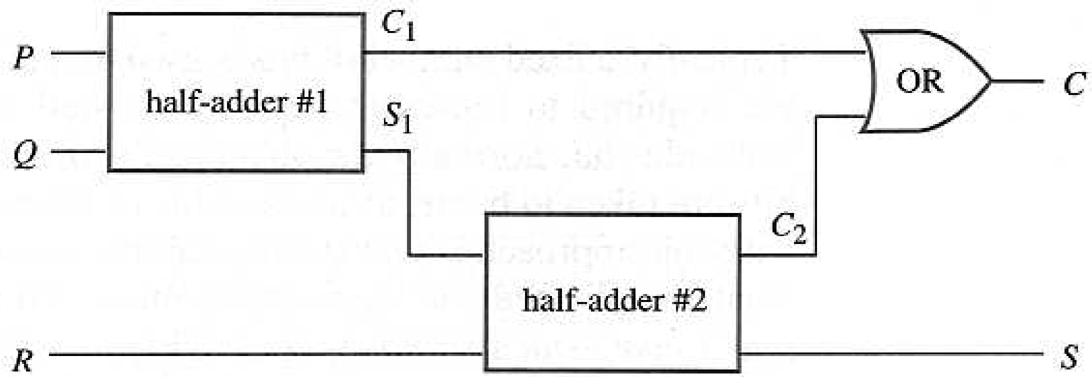
The half-adder.

When adding binary bits, we have the following (in base-2)

$$\begin{array}{ccccc} 1 & + & 1 & = & 10 \\ 1 & + & 0 & = & 01 \\ 0 & + & 1 & = & 01 \\ 0 & + & 0 & = & 00 \end{array}$$

FULL-ADDER

Circuit



Input/Output Table

P	Q	R	C	S
1	1	1	1	1
1	1	0	1	0
1	0	1	1	0
1	0	0	0	1
0	1	1	1	0
0	1	0	0	1
0	0	1	0	1
0	0	0	0	0

Homework #1 — Due 9/15/2006, 12:00pm, GMCS-587

Read sections 1.4 and 1.5 for background and entertainment value.

(**Epp-1.4.26, Epp-1.4.28** — Suggested, but not due.)

Epp-1.3: 13, 21, 39

Epp-1.3: Read examples 1.3.15, 1.3.16

Epp-1.2: 13, 24, 25, 26, 27

Epp-1.1: 3, 14, 16, 21, 25, 29, 31, 37, 41

Extra Brain-Twister (for fun): ***Epp-1.1.54***

conjunction, **n.**, a complex sentence in logic true if and only if each of its components is true.

disjunction, **n.**, a compound sentence in logic formed by joining two simple statements by ***or***.

syllogism, **n.**, a deductive scheme of a formal argument consisting of a major and a minor premise and a conclusion (as in "every virtue is laudable; kindness is a virtue; therefore kindness is laudable.")

Homework: 3rd Edition $\leftrightarrow$ 2nd Edition

3rd Edition	2nd Edition
Problems	
1.1: 3, 14, 16, 21, 25, 29, 31, 37, 41	***1.1:*** 3, 12, 14, 19, 23, 27, 29, 35, 37
1.2: 13, 24, 25, 26, 27	***1.2:*** 13, 20, 21, 22, 23
1.3: 13, 21, 39	***1.3:*** 12, 20, 38
1.4: 26, 28	***1.4:*** 26, 28
Examples	
1.3.8	1.3.8
1.3.15, 1.3.16	1.3.14, 1.3.15

Please use the ***3rd Edition*** numbering when handing in your solutions.