

# Math 254: Introduction to Linear Algebra

## Notes #1.1 — Linear Equations

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# Outline

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  - SLOs: Linear Equations
  - Numbering of Lecture Notes
- 2 Linear Equations
  - Example — Finding the Unique Solution
  - A Case with Infinitely Many Solutions
  - A Case with No Solution
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## SLOs 1.1

## Linear Equations

After this lecture you should:

- Be able to *Interpret* solutions of linear systems of 3 variables as intersections of planes.
- *Know* that linear systems can have
  - 0 no,
  - 1 one, or
  - $\infty$  infinitely manysolutions.
- Be able to *Perform* basic *Row-Operations* to determine all solutions of linear systems.

## Why are the Notes Numbered the Way They Are???

The numbering of the topics originates from the the structure of Otto Bretcher's book (used Fall 2015 — Fall 2016).

Since adopting Gilbert Strang's book (Spring 2017 — Spring 2018) the main “chapter topic number” (the “1” in  $1.n$ ) is retained, but the “section number” the (“1” in  $n.1$ ) is slowly being replaced by the the enumerated lecture number on each topic.

This is the **first** lecture on the **First** topic (Linear Equations); hence Notes #**1.1**.

References to the particular sections of Gilbert Strang's book will be added in the form [GS5-§1.1] (meaning section 1.1 in the 5th edition).

## A First Example

We sweep the history lessons under our infinitely stretchable rug, and focus on the system of linear equations:

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 39 \\ x & + & 3y & + & 2z & = & 34 \\ 3x & + & 2y & + & z & = & 26 \end{array} \right| \begin{array}{ll} r_1 & \text{"row \#1"} \\ r_2 & \vdots \\ r_3 & \vdots \end{array}$$

and the question:

— **What values of  $x$ ,  $y$ , and  $z$  satisfy this system?**

## Manipulate the System to Find the Solution

We want to manipulate the system

$$\text{From } \begin{cases} x + 2y + 3z = 39 \\ x + 3y + 2z = 34 \\ 3x + 2y + z = 26 \end{cases} \quad \text{To } \begin{cases} x = ??? \\ y = ??? \\ z = ??? \end{cases}$$

in order to reveal the solution.

But, before we do that, let's discuss a Graphical / Geometric Interpretation of the system...

→→→ **Quick Geometric “Detour!”** →→→

## Geometric Interpretation

If we view each row/equation in the system

$$\begin{cases} x + 2y + 3z = 39 \\ x + 3y + 2z = 34 \\ 3x + 2y + z = 26 \end{cases}$$

as a plane in 3D ( $x$ - $y$ - $z$ ) space; the solution represents the point(s?) where the planes meet.

Solve for  $z$  in each one of the equations:

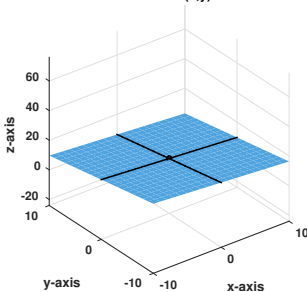
$$z_1(x, y) = (39 - x - 2y)/3$$

$$z_2(x, y) = (34 - x - 3y)/2$$

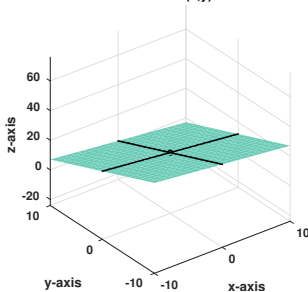
$$z_3(x, y) = (26 - 3x - 2y)$$

## The Three Planes

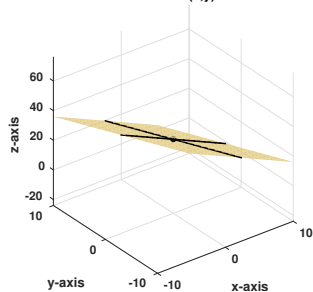
The Plane  $z_1(x,y)$



The Plane  $z_2(x,y)$



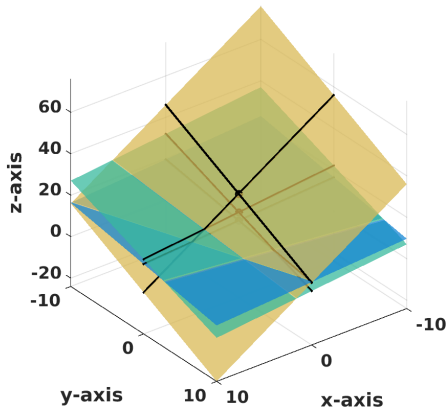
The Plane  $z_3(x,y)$



**Figure:** The planes  $z_1(x,y)$ ,  $z_2(x,y)$ , and  $z_3(x,y)$  visualized. As a reference, the black lines on the planes are aligned with the x- and y-axes.



## The Three Planes — Intersecting



**Figure:** The planes  $z_1(x, y)$ ,  $z_2(x, y)$ , and  $z_3(x, y)$  visualized. We are looking for the (in this case) **ONE POINT** where they all meet. [Movie]

## OK, Back to Solving the System

We can

- **ADD** or **SUBTRACT** multiples (fractions) of any equation (row) to/from another equation; or
- **MULTIPLY / DIVIDE / SCALE** an equation,

**without changing the solution.**

We do this in order to successively *Eliminate* variables from the equations, so that in the end we reveal the solution.

[GS5-§2.2 — “THE IDEA OF ELIMINATION”]

# Solving ...

“Forward elimination”  $\rightsquigarrow$  Upper Triangular System

“Forward elimination” stage:

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 39 \\ x & + & 3y & + & 2z & = & 34 \\ 3x & + & 2y & + & z & = & 26 \end{array} \right| \begin{array}{l} \text{subtract } r_1 \\ \text{subtract } 3r_1 \end{array}$$

we get:

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 39 \\ & & y & - & z & = & -5 \\ & - & 4y & - & 8z & = & -91 \end{array} \right| \text{add } 4r_2$$

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 39 \\ & & y & - & z & = & -5 \\ & & & - & 12z & = & -111 \end{array} \right| \text{divide by } (-12)$$

## ... Solution

“Back-substitution”  $\rightsquigarrow$  The Answer

“Backward elimination” or “Back-substitution” starts...

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 39 \\ & & y & - & z & = & -5 \\ & & & & z & = & 9.25 \end{array} \right| \begin{array}{l} \text{subtract } 3r_3 \\ \text{add } r_3 \end{array}$$

$$\left| \begin{array}{rrcr} x & + & 2y & = & 11.25 \\ & & y & = & 4.25 \\ & & & z & = & 9.25 \end{array} \right| \text{subtract } 2r_2$$

$$\left| \begin{array}{rrcr} x & & & = & 2.75 \\ & y & & = & 4.25 \\ & & z & = & 9.25 \end{array} \right|$$

## Check the Solution!

Plug

$$\begin{cases} x = 2.75 \\ y = 4.25 \\ z = 9.25 \end{cases}$$

into

$$\begin{cases} x + 2y + 3z = 39 \\ x + 3y + 2z = 34 \\ 3x + 2y + z = 26 \end{cases}$$

and see that

$$\begin{cases} 2.75 + 2 \times 4.25 + 3 \times 9.25 = 39 \\ 2.75 + 3 \times 4.25 + 2 \times 9.25 = 34 \\ 3 \times 2.75 + 2 \times 4.25 + 9.25 = 26 \end{cases}$$

- There are different paths to the solution:
  - for instance you can fully eliminate each column (“up” as well as “down”) before proceeding to the next one;  
in the end you will arrive at the same solution.
  - This strategy does not separate into “forward” and “backward” stage
- When you do things by “hand” you will probably pick a path (order of operations) which is unique to the problem, and which makes the algebra as simple as possible.

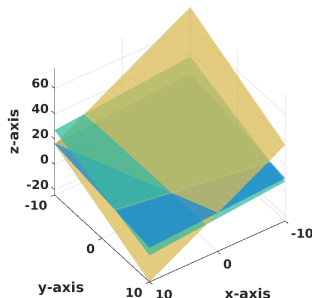
[“Live Math” starting from bottom of slide #11]



- When you do things by “code” (in software), you develop an algorithm (recipe / step-by-step instructions) which always follows the same path, since no algebra is “hard” for the computer (except dividing by zero... which is very very bad.)
  - The forward/backward split has the added benefit that collecting the results gives some useful “side products” — see [GS5-§2.6 — “ELIMINATION = FACTORIZATION;  $A = LU$ ”]

## What About the Geometric Interpretation?!?

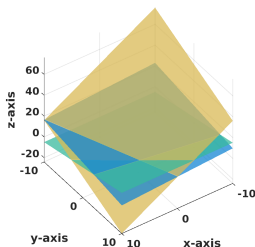
In a lot of books the Geometric Interpretation “lives” as its own separate “thing,” but it is natural to wonder what the impact of the solution procedure (Elimination) has on the Geometry.



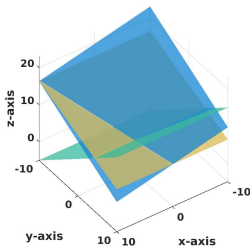
$$\begin{cases} x + 2y + 3z = 39 \\ x + 3y + 2z = 34 \\ 3x + 2y + z = 26 \end{cases}$$



## Eliminate $x$ from Equations 2, and 3



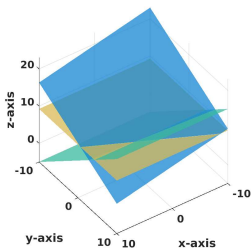
$$\begin{cases} x + 2y + 3z = 39 \\ y - z = -5 \\ 3x + 2y + z = 26 \end{cases}$$



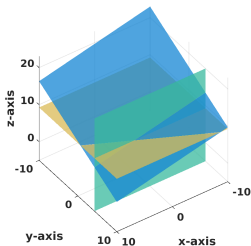
$$\begin{cases} x + 2y + 3z = 39 \\ y - z = -5 \\ -4y - 8z = -91 \end{cases}$$

Note: the new  $z_2(x, y) = y + 5$ ,  
 $z_3(x, y) = (91 - 4y)/8$  do not depend on  $x$ .

Eliminate  $y$  from Equation 3; and  $z$  from Equation 2

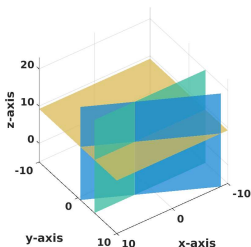


$$\begin{cases} x + 2y + 3z = 39 \\ y - z = -5 \\ z = 9.25 \end{cases}$$

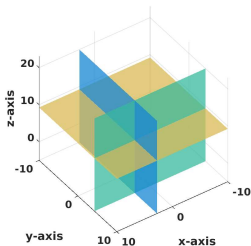


$$\begin{cases} x + 2y + 3z = 39 \\ y = 4.25 \\ z = 9.25 \end{cases}$$

# Eliminate $z$ from Equation 1; and $y$ from Equation 1



$$\begin{cases} x + 2y = 11.25 \\ y = 4.25 \\ z = 9.25 \end{cases}$$



$$\begin{cases} x = 2.75 \\ y = 4.25 \\ z = 9.25 \end{cases}$$

## Orthogonal Planes $\rightsquigarrow$ Easy to “read” the Solution

Looking the sequence of intersection planes as we go through the elimination steps:

We realize that our **Geometric Goal** was to create **Orthogonal Planes** intersecting in the solution point.

The idea of *Orthogonalization* will show up in various contexts; but it is always a tool which makes it easy (well, *easier*) to identify the property we are after. — In this particular case the solution to a linear system.

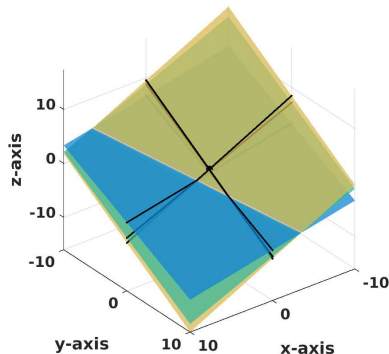
## What Can Happen?

A Linear System can have

- a unique solution (like our previous example)
- infinitely many solutions
- no solution

More solutions than you can shake an infinite stick at...

$$\begin{cases} 2x + 4y + 6z = 0 \\ 4x + 5y + 6z = 3 \\ 7x + 8y + 9z = 6 \end{cases}$$



**Figure:** We sense trouble when the three planes meet not in one point, but along a common line in space. Let us see how that manifests itself in the elimination process.

## Looking for infinitely many solutions...

$$\left| \begin{array}{rrcr} 2x & + & 4y & + & 6z & = & 0 \\ 4x & + & 5y & + & 6z & = & 3 \\ 7x & + & 8y & + & 9z & = & 6 \end{array} \right| \quad \begin{array}{l} \text{divide by 2} \end{array}$$

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 0 \\ 4x & + & 5y & + & 6z & = & 3 \\ 7x & + & 8y & + & 9z & = & 6 \end{array} \right| \quad \begin{array}{l} -4r_1 \\ -7r_1 \end{array}$$

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 0 \\ & - & 3y & - & 6z & = & 3 \\ & - & 6y & - & 12z & = & 6 \end{array} \right| \quad \begin{array}{l} \text{divide by } -3 \\ \text{divide by } -6 \end{array}$$

Looking for infinitely many solutions...

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 0 \\ & & y & + & 2z & = & -1 \\ & & y & + & 2z & = & -1 \end{array} \right| \quad -r_2$$

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 0 \\ & & y & + & 2z & = & -1 \\ & & & & 0 & = & 0 \end{array} \right| \quad \begin{array}{l} -2r_2 \\ \\ \text{d'oh!} \end{array}$$

$$\left| \begin{array}{rrcr} x & & & - & z & = & 2 \\ & & y & + & 2z & = & -1 \\ & & & & 0 & = & 0 \end{array} \right|$$



... Oh, there they are!

We have:

$$\begin{cases} x - z = 2 \\ y + 2z = -1 \\ 0 = 0 \end{cases}$$

This means that

$$\begin{cases} x = 2 + z \\ y = -1 - 2z \end{cases}$$

describes the line in space where the planes intersect;  $x$  and  $y$  are given as functions of  $z$ ; the line is  $(2 + z, -1 - 2z, z)$ ,  $z \in [-\infty, \infty]$ .

We introduce a **parameter** (or “alias”)  $z = t$ , which allows us to write

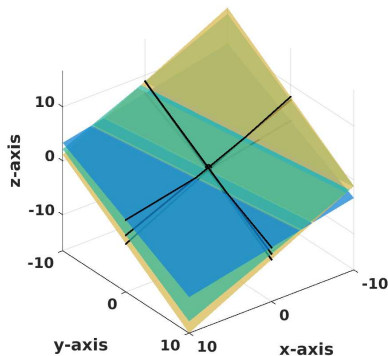
$$\begin{cases} x = 2 + t \\ y = -1 - 2t \\ z = t \end{cases}$$

We can write this in “line format” as:

$$(x, y, z) = (2, -1, 0) + t(1, -2, 1), t \in [-\infty, \infty].$$

Somebody ran away with the solution!

$$\begin{cases} x + 2y + 3z = 0 \\ 4x + 5y + 6z = 3 \\ 7x + 8y + 9z = 0 \end{cases}$$



**Figure:** Here, the planes intersect (pair-wise), but not in any common point, or line. Again, we go through the elimination to see how this manifests itself in the computation.

## Looking for Nothing...

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 0 \\ 4x & + & 5y & + & 6z & = & 3 \\ 7x & + & 8y & + & 9z & = & 0 \end{array} \right| \begin{array}{l} \\ -4r_1 \\ -7r_1 \end{array}$$

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 0 \\ & - & 3y & - & 6z & = & 3 \\ & - & 6y & - & 12z & = & 0 \end{array} \right| \begin{array}{l} \\ \text{divide by } -3 \\ \text{divide by } -6 \end{array}$$

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 0 \\ & & y & + & 2z & = & -1 \\ & & y & + & 2z & = & 0 \end{array} \right| \begin{array}{l} \\ \\ -r_2 \end{array}$$

## Looking for Nothing... and Finding Nonsense!

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 0 \\ & & y & + & 2z & = & -1 \\ & & & & \mathbf{0} & = & \mathbf{1} \end{array} \right| \quad \text{Say "What?!?"}$$

This system of equations is said to be *inconsistent*, and no solutions exist.

### End-of-Problem

#### However:

It is still possible to find the pair-wise intersections of the planes. Since the first 2 equations are the same as in the previous  $\infty$ -many solutions example; one such line intersection is  $(x, y, z) = (2, -1, 0) + t(1, -2, 1)$ ,  $t \in [-\infty, \infty]$ .

## Finding More Pair-Wise Intersections (#2)

### Equations #1 and #3

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 0 \\ 7x & + & 8y & + & 9z & = & 0 \end{array} \right| \quad -7r_1$$

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 0 \\ & - & 6y & - & 12z & = & 0 \end{array} \right| \quad \text{divide by } -6$$

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 0 \\ & & y & + & 2z & = & 0 \end{array} \right| \quad -2r_2$$

$$\left| \begin{array}{rrcr} x & & & - & z & = & 0 \\ & & y & + & 2z & = & 0 \end{array} \right|$$

Giving the line  $(x, y, z) = (0, 0, 0) + t(1, -2, 1)$ ,  $t \in [-\infty, \infty]$ .

## Finding More Pair-Wise Intersections (#3)

### Equations #2 and #3

$$\left| \begin{array}{rrcr} 4x & + & 5y & + & 6z & = & 3 \\ 7x & + & 8y & + & 9z & = & 0 \end{array} \right| \quad \begin{array}{l} \frac{1}{4} \\ \frac{1}{7} \end{array}$$

$$\left| \begin{array}{rrcr} x & + & \frac{5}{4}y & + & \frac{6}{4}z & = & \frac{3}{4} \\ x & + & \frac{8}{7}y & + & \frac{9}{7}z & = & 0 \end{array} \right| \quad -r_1$$

$$\left| \begin{array}{rrcr} x & + & \frac{5}{4}y & + & \frac{6}{4}z & = & \frac{3}{4} \\ & - & \frac{3}{28}y & - & \frac{3}{14}z & = & -\frac{3}{4} \end{array} \right| \quad +\frac{35}{3}r_2$$

## Finding More Pair-Wise Intersections (#3)

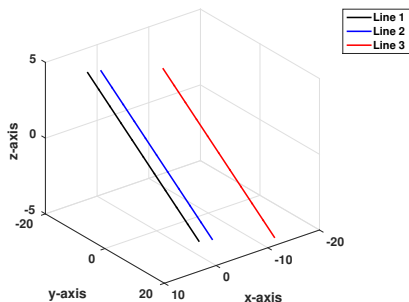
$$\begin{array}{l} \left| \begin{array}{rrcr} x & & - & z & = & -8 \\ & - & \frac{3}{28}y & - & \frac{3}{14}z & = & -\frac{3}{4} \end{array} \right| & -\frac{28}{3} \\ \left| \begin{array}{rrcr} x & & - & z & = & -8 \\ & y & + & 2z & = & 7 \end{array} \right| \end{array}$$

Giving the line  $(x, y, z) = (-8, 7, 0) + t(1, -2, 1)$ ,  $t \in [-\infty, \infty]$ .

Leaving us with 3 lines in space

$$\begin{aligned} (x, y, z)_1 &= (2, -1, 0) + t(1, -2, 1) \\ (x, y, z)_2 &= (0, 0, 0) + t(1, -2, 1) \\ (x, y, z)_3 &= (-8, 7, 0) + t(1, -2, 1) \end{aligned}$$

## Pair-Wise Intersections — Visualized



$$\begin{aligned}(x, y, z)_1 &= (2, -1, 0) + t(1, -2, 1) \\(x, y, z)_2 &= (0, 0, 0) + t(1, -2, 1) \\(x, y, z)_3 &= (-8, 7, 0) + t(1, -2, 1)\end{aligned}$$

**Figure:** The lines are parallel, and never intersect.

OK, we have squeezed the last “juice” out of this problem...



## Suggested Problems 1.1

## Available on “Learning Glass” videos:

- (1.1.1) Find all solutions to a 2-by-2 linear system using elimination.
- (1.1.3) Find all solutions to a 2-by-2 linear system using elimination.
- (1.1.7) Find all solutions to a 3-by-3 linear system using elimination.
- (1.1.14) Find all solutions to a 3-by-3 linear system using elimination.
- (1.1.19) Find all solutions to a 3-by-3 linear system *with a parameter  $k$*  using elimination. For what value(s) of  $k$  do we have one / no / infinitely many solutions?
- (1.1.21) Find three numbers, given then sums of all the pairs,
- (1.1.42) Solve upper and lower triangular linear systems.

## Metacognitive Exercise — Thinking About Thinking & Learning

I know / learned	Almost there	Huh?!?
Right After Lecture		
After Thinking / Office Hours / SI-session		
After Reviewing for Quiz/Midterm/Final		

(1.1.1), (1.1.3)

**(1.1.1)** Find all solutions of the linear system using elimination; check your solution.

$$\left| \begin{array}{rcrcrcrcl} x & + & 2y & = & 1 \\ 2x & + & 3y & = & 1 \end{array} \right|$$

**(1.1.3)** Find all solutions of the linear system using elimination; check your solution.

$$\left| \begin{array}{rcrcrcrcl} 2x & + & 4y & = & 3 \\ 3x & + & 6y & = & 2 \end{array} \right|$$

(1.1.7), (1.1.14)

**(1.1.7)** Find all solutions of the linear system using elimination; check your solution.

$$\begin{cases} x + 2y + 3z = 1 \\ x + 3y + 4z = 3 \\ x + 4y + 5z = 4 \end{cases}$$

**(1.1.14)** Find all solutions of the linear system.

$$\begin{cases} x + 4y + z = 0 \\ 4x + 13y + 7z = 0 \\ 7x + 22y + 13z = 1 \end{cases}$$

(1.1.19), (1.1.21)

**(1.1.19)** Consider the linear system

$$\begin{cases} x + y - z = -2 \\ 3x - 5y + 13z = 18 \\ x - 2y + 5z = k \end{cases}$$

where  $k$  is an arbitrary number.

- For which values of  $k$  does this system have one. or infinitely many solutions?
- For each value of  $k$  you found in part a, how many solutions does the system have.
- Find all solutions for each value of  $k$ .

**(1.1.21)** The sums of any two of three real numbers are 24, 28, and 30. Find these numbers.

(1.1.42)

**(1.1.42)** Linear systems are particularly easy to solve when they are in triangular form (*i.e.* all entries above or below the diagonal are zero).

a. Solve the lower triangular system

$$\begin{array}{ccccccccc} x_1 & & & & & & & & & = & -3 \\ -3x_1 & + & x_2 & & & & & & & = & 14 \\ x_1 & + & 2x_2 & + & x_3 & & & & & = & 9 \\ -x_1 & + & 8x_2 & - & 5x_3 & + & x_4 & & & = & 33 \end{array}$$

b. Solve the upper triangular system

$$\begin{array}{ccccccccc} x_1 & + & 2x_2 & - & x_3 & + & 4x_4 & = & -3 \\ & & x_2 & + & 3x_3 & + & 7x_4 & = & 5 \\ & & & & x_3 & + & 2x_4 & = & 2 \\ & & & & & & x_4 & = & 0 \end{array}$$

## Row-Reductions Revisited

“Forward elimination”  $\rightsquigarrow$  Upper Triangular System

“Forward elimination” stage:

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 39 \\ x & + & 3y & + & 2z & = & 34 \\ 3x & + & 2y & + & z & = & 26 \end{array} \right| \begin{array}{l} \text{subtract } r_1 \\ \text{subtract } 3r_1 \end{array}$$

**Details** (subtracting  $r_1$  from  $r_2$ ):

$$\begin{array}{r|rrcr} (-1) & x & + & 2y & + & 3z & = & 39 \\ (+1) & x & + & 3y & + & 2z & = & 34 \\ \hline = & & & y & - & z & = & -5 & \text{new } r_2 \end{array}$$

**Details** (subtracting  $3r_1$  from  $r_3$ ):

$$\begin{array}{r|rrcr} (-3) & x & + & 2y & + & 3z & = & 39 \\ (+1) & 3x & + & 2y & + & z & = & 26 \\ \hline = & & & -4y & - & 8z & = & -91 & \text{new } r_3 \end{array}$$

## Row-Reductions Revisited

“Forward elimination”  $\rightsquigarrow$  Upper Triangular System

After elimination of  $x$  from  $r_2$  and  $r_3$ , we have:

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 39 \\ & & y & - & z & = & -5 \\ & - & 4y & - & 8z & = & -91 \end{array} \right| \quad \begin{array}{l} \text{add } 4r_2 \end{array}$$

**Details** (adding  $4r_2$  to  $r_3$ ):

$$\begin{array}{r|rrcr} (+4) & & y & - & z & = & -5 \\ (+1) & - & 4y & - & 8z & = & -91 \\ \hline = & & & - & 12z & = & -111 & \text{new } r_3 \end{array}$$



## Row-Reductions Revisited

“Forward elimination”  $\rightsquigarrow$  Upper Triangular System

The system is now in “upper triangular” form; next we make the coefficient of  $z$  (in  $r_3$ ) one (1):

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 39 \\ & & y & - & z & = & -5 \\ & & & - & 12z & = & -111 \end{array} \right| \quad \text{divide by } (-12)$$

**Details** (dividing  $r_3$  by  $(-12)$ ):

$$\begin{array}{r|rrcr} (\times \frac{1}{12}) & & - & 12z & = & -111 \\ \hline = & & & z & = & \frac{111}{12} & \text{new } r_3 \end{array}$$

... and  $\frac{111}{12} = 9.25$ .

## Row-Reductions Revisited

“Back-substitution”  $\rightsquigarrow$  The Answer

“Backward elimination” or “Back-substitution” starts...

$$\left| \begin{array}{rrcr} x & + & 2y & + & 3z & = & 39 \\ & & y & - & z & = & -5 \\ & & & & z & = & 9.25 \end{array} \right| \begin{array}{l} \text{subtract } 3r_3 \\ \text{add } r_3 \end{array}$$

**Details** (adding  $r_3$  to  $r_2$ ):

$$\begin{array}{r|rrcr} (+1) & & y & - & z & = & -5 \\ (+1) & & & & z & = & 9.25 \\ \hline = & & y & & & = & 4.25 & \text{new } r_2 \end{array}$$

**Details** (subtracting  $3r_3$  from  $r_1$ ):

$$\begin{array}{r|rrcr} (+1) & x & + & 2y & + & 3z & = & 39 \\ (-3) & & & & & z & = & 9.25 \\ \hline = & x & + & 2y & & & = & 11.25 & \text{new } r_1 \end{array}$$

## Row-Reductions Revisited

“Back-substitution”  $\rightsquigarrow$  The Answer

Lastly, we eliminate  $y$  from  $r_1$ :

$$\left| \begin{array}{rcl} x + 2y & = & 11.25 \\ & y & = 4.25 \\ & & z = 9.25 \end{array} \right| \begin{array}{l} \text{subtract } 2r_2 \\ \end{array}$$

**Details** (subtracting  $2r_2$  from  $r_1$ ):

$$\begin{array}{r|lcl} (+1) & x + 2y & = & 11.25 \\ (-2) & & y & = 4.25 \\ \hline = & x & = & 2.75 \quad \text{new } r_1 \end{array}$$

We arrive at:

$$\left| \begin{array}{rcl} x & = & 2.75 \\ & y & = 4.25 \\ & & z = 9.25 \end{array} \right|$$

Online Labs —  $\langle \text{whine} \rangle \text{Why}??? \langle / \text{whine} \rangle$ 

[New Fall 2019]

In an effort to give you another point of view of the material, the development of online "labs" and/or "demonstrations" has started. Since **python** is the computer language *du jour*, that is the choice. Everything is presented in terms of **jupyter notebooks**.

How Much is THIS going to cost??? — the Python language itself, and the Jupyter notebook environment can be downloaded and installed for the staggering cost of **\$0** (with a full money-back guarantee).

All the necessary components: **Python**, **Pip**, **Anaconda**, and **Jupyter** are available for {Linux, Windows, MacOSX}

Wait. What? I have to download and install stuff?!? *C'mon, maaaaaan, this is a MATH class!!!* — R.E.L.A.X., there are multiple options for viewing and running the notebooks online, aka "in the cloud" (aka "on somebody elses hardware") without installing anything.

## Lab 1.1, Getting Started

[c'mon. give it a go!]

All lab materials can be found at <http://terminus.SDSU.EDU/SDSU/Math254/?r=labs>

At the start of Fall 2019, there are only two labs — 1.1, and 1.2.

There are two “modes” to view the labs:

### • Non-Interactive

- Just click the lab you want to view under the heading “**Static Views**”; the lab will open up using [nbviewer.jupyter.org](http://nbviewer.jupyter.org) — no need to install anything.

### • Interactive

- 0 If you want to “play” with the lab (change the numbers, test other things)...
- 1 First download (right-click + save) the lab under the heading “**Source Link**”
- 2 Then you can upload/view/interact with the lab on e.g.:
  - <https://colab.research.google.com/>
  - <https://mybinder.org/>
  - Your local Python / Jupyter notebook installation

## Ask “Uncle Google”!

There are (probably) other free services that will allow you to run Jupyter notebooks “in the cloud.”

As these services change/are updated often, it is not worth trying to document HOW to use them here. The best thing to do it is just give it a try; read the help/instructions available on e.g. [mybinder.org](https://mybinder.org) or [colab.research.google.com](https://colab.research.google.com).

You favorite uncle — “Uncle Google” — is always ready to help:

- “how do I run jupyter notebook on mybinder”
- “how do I run jupyter notebook on colab”
- “how do I install X on Y”,
  - where  $X \in \{\text{python, jupyter, ...}\}$ , and
  - $Y \in \{\text{Windows, MacOS, Fedora, Ubuntu, Debian, ...}\}$