Math 254: Introduction to Linear Algebra

Notes #1.2 — Matrices, Vectors, ...

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Outline

1. Student Learning Objectives
   - SLOs: Matrices, Vectors, ...
2. Matrices, Vectors; Gauss-Jordan Elimination
   - Matrix – Vector Notation
   - Back to Solving Linear Systems
   - Summarizing
3. Suggested Problems
   - Suggested Problems
   - Lecture – Book Roadmap
4. Supplemental Material
   - Metacognitive Reflection
   - Problem Statements 1.2

SLOs 1.2 Matrices, Vectors, ...

After this lecture you should:

- Know basic language and concepts:
  - Matrices, vectors, and their components
  - Matrix types: square, diagonal, triangular, zero, identity
  - The collection of all *n*-vectors, denoted $\mathbb{R}^n$ is a vector space
- Vector addition
- Know the difference between the Coefficient matrix, and the Augmented matrix; their uses in the solution of linear systems
- Know how to use elimination to identify leading and non-leading (a.k.a. *free variables*), and when necessary introduce parameters to express *all* solutions of linear systems.
- Know what **Reduced–Row–Echelon–Form** (RREF) of a Matrix is, and how to achieve it using elementary row operations.

Equation Manipulation to Isolate Variables

Our current "business" is manipulating linear systems, of the form

$$\left| \begin{array}{rcrcrcr} 3x & + & 21y & - & 3z & = & 0 \\ -6x & - & 2y & - & z & = & 62 \\ 2x & - & 3y & + & 8z & = & 32 \end{array} \right|$$

into a form which reveals the values of x, y, and z:

$$\left| \begin{array}{ccccr} x & & & = & -3574/281 \\ & y & & = & 844/281 \\ & & z & = & 2334/281 \end{array} \right| .$$

We achieve this by cleverly adding/subtracting rows (equations) from each other.

Matrix Notation — "Encoding" the Information

We realize that all the important information is in the coefficients (numbers), and that the variables (x, y, z) just get carried around. We can "encode" all the information about the linear system

$$\left| \begin{array}{rcrcrcr} 3x & + & 21y & - & 3z & = & 0 \\ -6x & - & 2y & - & z & = & 62 \\ 2x & - & 3y & + & 8z & = & 32 \end{array} \right|$$

in a **matrix**

$$\underbrace{\begin{bmatrix} 3 & 21 & -3 & 0 \\ -6 & -2 & -1 & 62 \\ 2 & -3 & 8 & 32 \end{bmatrix}}_{\text{Augmented Matrix}}, \text{ or, sometimes: } \underbrace{\left[\begin{array}{rrr|r} 3 & 21 & -3 & 0 \\ -6 & -2 & -1 & 62 \\ 2 & -3 & 8 & 32 \end{array}\right]}.$$

Augmented Matrix with **Coefficient Matrix** and right-hand-side "separated."

Row – Column Indexing

Ponder the matrix "A" with 3 rows, and 4 columns:

$$A = \begin{bmatrix} 3 & 21 & -3 & 0 \\ -6 & -2 & -1 & 62 \\ 2 & -3 & 8 & 32 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

that is, usually we refer to the entries of a matrix A (upper-case), using double subscripts a_{ij} (lower-case); the subscripts i, and j are "standard" but r (row) and c (column) would be more intuitive.

Sometimes you see the notation $A \in \mathbb{R}^{3\times 4}$ to denote a 3-by-4 (always [Rows-by-Columns]) matrix where the entries are real ($\mathbb{R}$) numbers.

Note: The entries can be other mathematical objects, *e.g.* complex numbers, $\mathbb{C}$, polynomials, etc... but we will work with $\mathbb{R}$ for quite while.

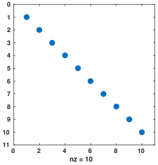
[Focus :: CS] What is a Matrix?

From a computer science point-of-view a matrix can be viewed as *data structure*, and depending on your mood (and/or preference of programming paradigm) you can think of it as *e.g.* a

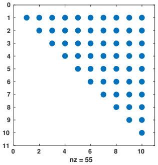
- C—C++ style *2-dimensional array*,
  - :: `double A[3][3]; /* A is a 3-by-3 matrix */`
  - :: `A[0][0] = 1;     /* Assigning 1 to a11 */`
  - :: `A[2][2] = 14;    /* Assigning 14 to a33 */`
  - :: Yes, some languages count from 0 to (n-1); others from 1 to n.
- or an abstract *container class*.
- Python
  - :: uses `(...)` for immutable "tuples" and `[...]` for "lists"...
  - :: a matrix is a lists-of-lists: `[[...], ..., [...]]`

Types of Matrices

- When $A \in \mathbb{R}^{n \times n}$, *i.e.* the matrix has the same number of rows and columns, it is a **square matrix**



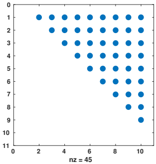
- A matrix is **diagonal** if all entries $a_{ij} = 0$ for all $i \neq j$. (Only entries of the type a_{ii} are non-zero.



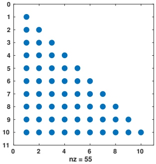
- A square matrix is **upper triangular** if all entries $a_{ij} = 0$ for all $i > j$.

Types of Matrices

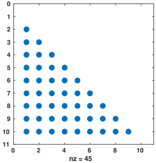
- A square matrix is **strictly upper triangular** if all entries $a_{ij} = 0$ for all $i \geq j$.



- A square matrix is **lower triangular** if all entries $a_{ij} = 0$ for all $i < j$.



- A square matrix is **strictly lower triangular** if all entries $a_{ij} = 0$ for all $i \leq j$.



Types of Matrices

- A matrix where all entries are zero is (surprisingly?) called a **zero matrix**.
- A square matrix where all diagonal entries are *ones*, and the off-diagonal entries are *zeros*

$$I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

is called an **identity matrix**. " $\cdots$ " and " $\vdots$ " denote padding with 0-entries, and " $\ddots$ " diagonal 1-entries; filling the matrix out to its full size (whatever that may be).

Matrices of size $n \times 1$ and $1 \times n \Rightarrow$ "Vectors"

- A "matrix" with only one column is called a **column vector**:

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

- A "matrix" with only one row is called a **row vector**:

$$\vec{w}^T = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix}$$

By mathematical convention a **vector** is a **column vector**; so $\vec{v}$ and $\vec{w}$ are (column) vectors. The notation $\vec{w}^T$ is the *transpose* of the vector $\vec{w}$, which is a *row vector*.

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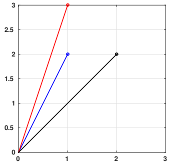
Vector "Language"

- The entries of a vector are called its **components**; v_k is the k^{th} component of the vector $\vec{v}$.
- The set (collection) of *all* (column) vectors with n components is denoted by $\mathbb{R}^n$; we refer to $\mathbb{R}^n$ as a **vector space**.

It is easy to visualize vectors in $\mathbb{R}^2$; we can think of the vector $\vec{v}$ as an arrow from the origin $(0,0)$ to the point $(x,y) = (v_1, v_2)$:



In the figure we have

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 2 \\ 2 \end{bmatrix},$$

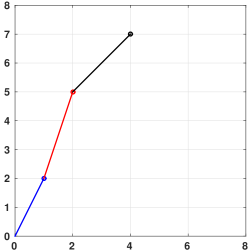
Without confusion, we can just let the terminal points $(x,y) = (v_1, v_2)$ represent the vectors.

Adding Vectors

With:

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 2 \\ 2 \end{bmatrix},$$

We can graphically show how to add vectors:



That is

$$\vec{u} + \vec{v} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \quad \vec{u} + \vec{v} + \vec{w} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}.$$

Making Use of our Matrix – Vector Notation

Now, given a linear system:

$$\left| \begin{array}{ccccccc} 2x & + & 8y & + & 4z & = & 2 \\ 2x & + & 5y & + & z & = & 5 \\ 4x & + & 10y & - & z & = & 1 \end{array} \right|$$

We can extract the **Coefficient Matrix** (containing the coefficients of the unknown variables in the system)

$$\begin{bmatrix} 2 & 8 & 4 \\ 2 & 5 & 1 \\ 4 & 10 & -1 \end{bmatrix},$$

or the **augmented matrix**

$$\begin{bmatrix} 2 & 8 & 4 & 2 \\ 2 & 5 & 1 & 5 \\ 4 & 10 & -1 & 1 \end{bmatrix},$$

which captures all the information in the linear system.

The Augmented Matrix

Often, we separate the coefficients from the right-hand-side information in the Augmented Matrix:

$$\begin{bmatrix} 2 & 8 & 4 & 2 \\ 2 & 5 & 1 & 5 \\ 4 & 10 & -1 & 1 \end{bmatrix} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 8 & 4 & 2 \\ 2 & 5 & 1 & 5 \\ 4 & 10 & -1 & 1 \end{array}\right]$$

Solving Linear Systems Using the Augmented Matrix

We can solve the linear system by manipulating the Augmented Matrix:

$$\left[\begin{array}{ccc|c} 2 & 8 & 4 & 2 \\ 2 & 5 & 1 & 5 \\ 4 & 10 & -1 & 1 \end{array}\right] \begin{array}{l} /\,2 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 2 & 5 & 1 & 5 \\ 4 & 10 & -1 & 1 \end{array}\right] \begin{array}{l} \\ -2r_1 \\ -4r_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 0 & -3 & -3 & 3 \\ 0 & -6 & -9 & -3 \end{array}\right] \begin{array}{l} \\ /\,(-3) \\ /\,(-3) \end{array}$$

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Solving Linear Systems Using the Augmented Matrix

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 3 & 1 \end{array}\right] \begin{array}{c} \\ \\ -2r_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{r} -2r_3 \\ -r_3 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 0 & -5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{c} -4r_2 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right], \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -4 \\ 3 \end{bmatrix}$$

The (Math) World is not limited to 2, or 3 Variables (Dimensions)

We can (easily?) imagine a system of 4 linear equations with 7 unknowns:

$$\left| \begin{array}{rrrrrrrrrrrrrr} x_1 & - & x_2 & & & & & + & 4x_5 & & & & 7777x_7 & = & 1 \\ & & & & x_3 & & & + & 2x_5 & & & - & 777x_7 & = & 2 \\ & & & & & & x_4 & & & & & & 77x_7 & = & 3 \\ & & & & & & & & & & x_6 & - & 7x_7 & = & 4 \end{array} \right|,$$

so that solving for the leading variables* gives:

$$\left| \begin{array}{rcl} x_1 & = & 1 + x_2 - 4x_5 - 7777x_7 \\ x_3 & = & 2 - 2x_5 + 777x_7 \\ x_4 & = & 3 - 77x_7 \\ x_6 & = & 4 + 7x_7 \end{array} \right|$$

* **Leading variables** are the first ones to appear in each equation (after elimination); here x_1, x_3, x_4, and x_6.

Infinitely Many Solutions

If we parameterize (the non-leading, or "free" variables): $\mathbf{x_2} = \mathbf{s}$, $\mathbf{x_5} = \mathbf{t}$, and $\mathbf{x_7} = \mathbf{u}$, we can write the infinitely many solutions:

$$\begin{bmatrix} x_1 \\ \mathbf{x_2} \\ x_3 \\ x_4 \\ \mathbf{x_5} \\ x_6 \\ \mathbf{x_7} \end{bmatrix} = \begin{bmatrix} 1 + s - 4t - 7777u \\ \mathbf{s} \\ 2 - 2t + 777u \\ 3 - 77u \\ \mathbf{t} \\ 4 + 7u \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \\ 0 \\ 4 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -4 \\ 0 \\ -2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} -7777 \\ 0 \\ 777 \\ -77 \\ 0 \\ 7 \\ 1 \end{bmatrix}$$

If we plug that into the original system of linear equations we see that it indeed is the (collection of) solution(s)!

Note that s, t, and u are allowed to take any values in $\mathbb{R}$ (independent of each other)...

What Makes a System "Easy" to Solve?

Three properties make a system "easy" to solve:

- P1 The leading coefficient in each equation is 1.
- P2 The leading variable in each equation does not appear in any other equation.
- P3 The leading variables appear in "natural order," with increasing indices as we go down the system: x_1, x_3, x_4, and x_6 as opposed to any other ordering.

If/when the system does not satisfy these properties, we use elimination to get there...

Another Example...

Keeping P1, P2, P3 in mind...

We go straight to the Augmented Matrix:

$$\left[\begin{array}{ccccc|c} 2 & 4 & -2 & 2 & 4 & 2 \\ 1 & 2 & -1 & 2 & 0 & 4 \\ 3 & 6 & -2 & 1 & 9 & 1 \\ 5 & 10 & -4 & 5 & 9 & 9 \end{array}\right] \quad \begin{array}{c} (/2) \\ \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -1 & 1 & 2 & 1 \\ 1 & 2 & -1 & 2 & 0 & 4 \\ 3 & 6 & -2 & 1 & 9 & 1 \\ 5 & 10 & -4 & 5 & 9 & 9 \end{array}\right] \quad \begin{array}{r} \\ -r_1 \\ -3r_1 \\ -5r_1 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & -1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & -2 & 3 & -2 \\ 0 & 0 & 1 & 0 & -1 & 4 \end{array}\right] \quad \begin{array}{r} -r_2 \\ \\ +2r_2 \\ \\ \end{array}$$

Another Example...

Keeping P1, P2, P3 in mind...

$$\left[\begin{array}{ccccc|c} 1 & 2 & -1 & 0 & 4 & -2 \\ 0 & 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & 0 & -1 & 4 \end{array}\right] \begin{array}{l} +r_3 \\ \\ \\ -r_3 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \rightsquigarrow r_3 \\ \rightsquigarrow r_2 \\ \\ \end{array}$$

$$\left[\begin{array}{ccccc|c} \textcircled{1} & 2 & 0 & 0 & 3 & 2 \\ & & \textcircled{1} & 0 & -1 & 4 \\ & & & \textcircled{1} & -2 & 3 \\ & & & & & \end{array}\right]$$

leading zeros suppressed for clarity

... Identifying the Solutions

x_1	x_2	x_3	x_4	x_5	value
(1)	2			3	2
		(1)		−1	4
			(1)	−2	3

So that:

$$\left| \begin{array}{lcl} x_1 & = & 2 - 2x_2 - 3x_5 \\ x_3 & = & 4 + x_5 \\ x_4 & = & 3 + 2x_5 \end{array} \right|, \quad \text{or}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 - 2s - 3t \\ s \\ 4 + t \\ 3 + 2t \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \\ 3 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3 \\ 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}.$$

Parameters: $x_2 = s$, $x_5 = t$

Solving a System of Linear Equations

"Elimination Strategy"

We go equation-by-equation from top to bottom (i runs from 1 to n):

- For the i^{th} equation: if the leading variable is x_j with non-zero coefficient c; `divide` the equation by c to make the leading coefficient 1.
- Eliminate x_j from all other equations.
- Go to the next equation.

"Exit Strategy"

- If we get *zero* = *nonzero* at any point; then there are no solutions. [STOP]
- If we complete without inconsistencies:
  - **rearrange** the equations so that the leading variables are in *"natural order"*
  - Solve each equation for the leading variable
  - Choose parameters for the **non-leading variables** [if there are any] (appropriate "alphabet soup")
  - Express leading variables using parameters.

Reduced Row Echelon Form

After elimination according to this strategy, the matrix is in:

Reduced Row Echelon Form

A Matrix is said to be in *Reduced Row Echelon Form* if it satisfies the following conditions:

1. If a row has non-zero entries, then the first non-zero entry is a 1, called *the leading 1* (or *pivot*) of this row.
2. If a column contains a leading 1, then all other entries in that column are 0. [Elimination is Complete]
3. If a row contains a leading 1, then each row above it contains a leading 1 further to the left. [Sorting of Rows]

The last condition implies that rows of 0's, if any, must appear at the bottom of the matrix.

Getting to Reduced Row Echelon Form

We get to *Reduced Row Echelon Form* by performing

Elementary Row Operations

- Divide a row by a non-zero scalar
- Subtract a multiple of a row from another row
- Swap two rows

This strategy of solving linear systems by reduction to Reduced Row Echelon Form is referred to as **Gaussian Elimination**, or **Gauss-Jordan Elimination**.

Gauss (1777–1855), Jordan (1842–1899); but the Chinese used it looooong before that.

"Gauss-Jordan Elimination" $\rightsquigarrow$ RREF
"Gaussian Elimination" $\rightsquigarrow$ REF (leading variables NOT 1's; *LU*-factorization)

Suggested Problems 1.2

Available on "Learning Glass" videos:

(1.2.1) Find all solutions to a 2-by-3 linear system using elimination.

(1.2.3) Find all solutions to a 1-by-3 linear system using elimination.

(1.2.9) Find all solutions to a 3-by-6 linear system using elimination.

(1.2.11) Find all solutions to a 4-by-4 linear system using elimination.

(1.2.18) Determine which matrices are in RREF.

(1.2.21) Find values of matrix entries so that the resulting matrix is in RREF.

Lecture – Book Roadmap

Lecture	Book, [GS5–]
1.1	§2.2
1.2	§1.1, §1.3, §2.1, §2.3

Metacognitive Exercise — Thinking About Thinking & Learning

I know / learned	Almost there	Huh?!?
Right After Lecture		
After Thinking / Office Hours / SI-session		
After Reviewing for Quiz/Midterm/Final		

(1.2.1), (1.2.3)

(1.2.1) Find all solutions to the linear system using elimination:

$$\left|\begin{array}{ccccccc} x & + & y & - & 2z & = & 5 \\ 2x & + & 3y & + & 4z & = & 2 \end{array}\right|$$

(1.2.3) Find all solutions to the linear system using elimination:

$$\left|\begin{array}{ccccccc} x & + & 2y & + & 3z & = & 4 \end{array}\right|$$

(1.2.9), (1.2.11)

(1.2.9) Find all solutions to the linear system using elimination:

$$\left|\begin{array}{ccccccccccccc} & & & & & & x_4 & + & 2x_5 & - & x_6 & = & 2 \\ x_1 & + & 2x_2 & & & & & + & x_5 & - & x_6 & = & 0 \\ x_1 & + & 2x_2 & + & 2x_3 & & & - & x_5 & + & x_6 & = & 2 \end{array}\right|$$

(1.2.11) Find all solutions to the linear system using elimination:

$$\left|\begin{array}{ccccccccc} x_1 & & & + & 2x_3 & + & 4x_4 & = & -8 \\ & & x_2 & - & 3x_3 & - & x_4 & = & 6 \\ 3x_1 & + & 4x_2 & - & 6x_3 & + & 8x_4 & = & 0 \\ & - & x_2 & + & 3x_3 & + & 4x_4 & = & -12 \end{array}\right|$$

(1.2.18)

(1.2.18) Determine which of the matrices are in Reduced Row Echelon Form:

$$\textbf{a.} \begin{bmatrix} 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \textbf{b.} \begin{bmatrix} 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\textbf{c.} \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} \qquad \textbf{d.} \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

(1.2.21)

(1.2.21) For which values of a, b, c, d, and e is the following matrix in reduced-row-echelon-form?

$$\begin{bmatrix} 1 & a & b & 3 & 0 & -2 \\ 0 & 0 & c & 1 & d & 3 \\ 0 & e & 0 & 0 & 1 & 1 \end{bmatrix}$$

Piazza Setup and Use, 1 of 3 Does not apply Fall 2021 [TECH::SETUP]

Two Separate Websites

piazza

https://piazza.com/

- **Piazza Account**
  - Mini-Quizzes
  - Access using app, or web interface

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Blackboard at San Diego State University

https://Canvas.SDSU.edu/

- **GRADEBOOK**

Piazza Setup and Use, 2 of 3 Does not apply Fall 2021 [TECH::SETUP]

Goal: Your Points in the GRADEBOOK!

How do we get there?

1. Set up a **Piazza Account**
   - You should have gotten an inviatation sent to your `[...]@sdsu.edu` email address; or go to
   - `https://piazza.com/sdsu/fall2021/math254blomgren`
   - It is FREE
   - Download the app from the iOS App Store, or Google Play
2. **If you want credit:** You can register multiple email addresses to a piazza account — one of them must match your email address in Canvas.
   - In Piazza go to `Account/Email Settings`

Piazza Setup and Use, 3 of 3 Does not apply Fall 2021 [TECH::SETUP]

Goal: Your Points in the GRADEBOOK!

How do we get there?

3. Scores will be transferred after EACH LECTURE
    - The transfer is manual, so there may be a bit of a delay.
4. *If something is not right, please let me know!* — Email me!

If you cannot participate in the use of Piazza (due to lack of a suitable device) — let me know, and we'll figure something out.