1	2	3	4	5	6	7	8	9
RedID								

Math 254
Midterm #1 Practice Test

Tools: Brain/Pen/Pencil/Eraser/Paper.
Rules: This is just a test-test; see below:

v 2017.9.26.0

I, ____________________________, understand that this is a practice exam; which is provided for adults and entertainment purposes only. Any resemblence to a real test is purely intentional, but not guaranteed.

____
Signature (REQUIRED to continue (yeah, that will be enforced?!?))

Suggestions:

- Present your solutions using standard notation in an easy-to-read format. It is your job to convince yourself you did the problem correctly...
- ***Your answers MUST logically follow from your calculations in order to be considered! ("Miracle solutions" ⇒ −zero points.)***
- If you are a computer science major, perform all calculations in the hexadecimal basis.

Note:

- There is a high probablility that the actual test will have slightly fewer questions.

Problem	Pts Possible	Pts Scored
1	000	000
2	000	000
3	000	000
4	000	000
5	000	000
6	000	000
7	000	000
8	000	000
9	000	000
10	000	000
11	000	000
12	000	000
13	000	000
14	000	000
15	000	000
Total	000	0/0 "Undefined"

1. (§1.1) Consider the linear system

$$\left|\begin{array}{rcrcrcr} x & + & y & - & z & = & -2 \\ 3x & - & 5y & + & 13z & = & 18 \\ x & - & 2y & + & 5z & = & k \end{array}\right|$$

where $k \in \mathbb{R}^n$ is an arbitrary number.

a. For which values of k does this system have one, or infinitely many solutions?

b. For each value of k you found in part a, how many solutions does the system have.

c. Find all solutions for each value of k.

2. (§1.1,§1.2)

a. Solve the lower triangular system

$$\left|\begin{array}{rcrcrcrcr} x_1 & & & & & & & = & -3 \\ -3x_1 & + & x_2 & & & & & = & 14 \\ x_1 & + & 2x_2 & + & x_3 & & & = & 9 \\ -x_1 & + & 8x_2 & - & 5x_3 & + & x_4 & = & 33 \end{array}\right|$$

b. Solve the upper triangular system

$$\left|\begin{array}{rcrcrcrcr} x_1 & + & 2x_2 & - & x_3 & + & 4x_4 & = & -3 \\ & & x_2 & + & 3x_3 & + & 7x_4 & = & 5 \\ & & & & x_3 & + & 2x_4 & = & 2 \\ & & & & & & x_4 & = & 0 \end{array}\right|$$

c. Solve the upper triangular system

$$\left|\begin{array}{rcrcrcrcr} x_1 & + & 2x_2 & - & x_3 & + & 4x_4 & = & -3 \\ & & x_2 & + & 3x_3 & + & 7x_4 & = & 5 \\ & & & & x_3 & + & 2x_4 & = & 2 \end{array}\right|$$

3. (§1.2) Find all solutions to the linear system using elimination:

$$\left|\begin{array}{rcrcrcrcr} x_1 & & & + & 2x_3 & + & 4x_4 & = & -8 \\ & & x_2 & - & 3x_3 & - & x_4 & = & 6 \\ 3x_1 & + & 4x_2 & - & 6x_3 & + & 8x_4 & = & 0 \\ & - & x_2 & + & 3x_3 & + & 4x_4 & = & -12 \end{array}\right|$$

4. (§1.3) The reduced-row-echelon-forms (RREF) of the augmented matrices of three systems are given. How many solutions does each system have?

$$\textbf{(a)} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right], \quad \textbf{(b)} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 6 \end{array}\right], \quad \textbf{(c)} \left[\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right].$$

5. (§1.3) Find the rank of

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

6. (§1.3) Find all vectors $\vec{x}$ such that $A\vec{x} = \vec{b}$, where

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

7. (§2.1) Consider the transformations from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by

$$\textbf{(a.)} \begin{array}{lcl} y_1 & = & 2x_2 \\ y_2 & = & x_2 + 2 \\ y_3 & = & 2x_2 \end{array}, \quad \textbf{(b.)} \begin{array}{lcl} y_1 & = & 2x_2 \\ y_2 & = & 3x_3 \\ y_3 & = & x_1 \end{array}, \quad \textbf{(c.)} \begin{array}{lcl} y_1 & = & x_2 - x_3 \\ y_2 & = & x_1 x_3 \\ y_3 & = & x_1 - x_2 \end{array},$$

which ones are **linear** transformations?

8. (§2.1) Consider the transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ with

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 7 \\ 11 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 6 \\ 9 \end{bmatrix}, \quad \text{and } T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -13 \\ 17 \end{bmatrix}.$$

Find the matrix A of $T : \mathbb{R}^3 \to \mathbb{R}^2$.

9. (§2.1) Give a geometric interpretation of the linear transformation defined by the matrix

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Show the effect of the transformation on the letter L, described by the two vectors

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 2 \end{bmatrix}.$$

Is the transformation invertible? Find the inverse if it exists, and interpret it geometrically.

10. (§2.2) Sketch the image of the "L," described by the two vectors

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 2 \end{bmatrix}.$$

under the linear transformation

$$T(\vec{x}) = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \vec{x}.$$

11. (§2.2) Let L be the line in $\mathbb{R}^3$ that consists of all scalar multiples of the vector $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$.

(a.) Find the orthogonal projection of the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. onto L.

(b.) Find the reflection of the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. about the line L.

12. (§2.2) Find the...

a. scaling matrix A that transforms $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ into $\begin{bmatrix} 8 \\ -4 \end{bmatrix}$

b. orthogonal projection matrix B that transforms $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$

c. rotation matrix C that transforms $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ into $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

d. shear matrix D that transforms $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} 7 \\ 3 \end{bmatrix}$

e. reflection matrix E that transforms $\begin{bmatrix} 7 \\ 1 \end{bmatrix}$ into $\begin{bmatrix} -5 \\ 5 \end{bmatrix}$

13. (§2.3) Compute (if possible) the matrix products

$$\textbf{(a.)}\ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \textbf{(b.)}\ \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \textbf{(c.)}\ \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

14. (§2.4) Decide whether the matrices are invertible; if they are, find the inverse(s).

$$\textbf{(a.)}\ \begin{bmatrix} 2 & 3 \\ 5 & 8 \end{bmatrix}, \quad \textbf{(b.)}\ \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}, \quad \textbf{(c.)}\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

15. (§2.4) Decide whether the linear transformations are invertible; if they are, find the inverse transformation(s).

$$\textbf{(a.)}\ \left| \begin{array}{rcrcr} y_1 & = & 3x_1 & + & 5x_2 \\ y_2 & = & 5x_1 & + & 8x_2 \end{array} \right|, \quad \textbf{(b.)}\ \left| \begin{array}{rcrcr} y_1 & = & x_1 & + & 2x_2 \\ y_2 & = & 4x_1 & + & 8x_2 \end{array} \right|$$