1	2	3	4	5	6	7	8	9
RedID								

Math 254, Spring 2017
Midterm #2 Practice Test

Tools: Brain/Pen/Pencil/Eraser/Paper.
Rules: This is just a test-test; see below:

v 2017.3.16.0

I, ____________________________, understand that this is a practice exam; which is provided for adults and entertainment purposes only. Any resemblence to a real test is purely intentional, but not guaranteed.

____
Signature (REQUIRED to continue (yeah, that will be enforced?!?))

Suggestions:

- Present your solutions using standard notation in an easy-to-read format. It is your job to yourself you did the problem correctly...
- ***Your answers MUST logically follow from your calculations in order to be considered! ("Miracle solutions" $\Rightarrow$ $-$zero points.)***
- If you are a computer science major, perform all calculations in the hexadecimal basis.

Note:

- There is a high probablility that the actual test will have slightly fewer questions.

Problem	Pts Possible	Pts Scored
1	000	
2	000	
3	000	
4	000	
5	000	
6	000	
7	000	
8	000	
9	000	
0xA	000	
0xB	000	
Total	000	

1. (§3.1) Find vectors that span the *kernel* of

$$A = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 3 & 4 & -6 & 8 \\ 0 & -1 & 3 & 4 \end{bmatrix}$$

2. (§3.1) Find vectors that span the *image* of

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

3. (§3.1, §3.2) *(i)* Are the column vectors in

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

linearly independent? Why/Why Not? — Give *(ii)* a "mechanical" (computational) answer, and *(iii)* a "theoretical" answer which does not require any computations.

4. (§2.4, §3.2, §3.3) *(i)* Determine whether the given vectors are linearly independent:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}.$$

(ii) If we use these vectors as the columns in a matrix A, will the matrix be invertible? *(iii)* Why/Why not? *(iv)* If the inverse exists, compute it.

5. (§3.2) Consider the 5×4 matrix

$$A = \begin{bmatrix} | & | & | & | \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{v}_4 \\ | & | & | & | \end{bmatrix},$$

we are told that the vector

$$\vec{n}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

is in the *kernel* of A. *(i)* Find a linear relation between the vectors; and *(ii)* write $\vec{v}_4$ as a linear combination of $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$.

6. (§3.3) Find the reduced row echelon form of A; then find a basis for $\text{im}(A)$, and a basis $\ker(A)$, where

$$A = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & -1 \\ 3 & 4 & -6 & 8 \\ 0 & -1 & 3 & 1 \end{bmatrix}.$$

7. (§3.3) Determine whether the following vectors form a basis of $\mathbb{R}^4$:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 2 \\ 4 \\ 8 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -2 \\ 4 \\ -8 \end{bmatrix}.$$

8. (§3.3) *(i)* Find a basis of the subspace, V of $\mathbb{R}^4$ defined by the equation

$$2x_1 - x_2 + 2x_3 + 4x_4 = 0.$$

then *(ii)* find a basis for the orthogonal complement $V^{\perp}$.

9. (§3.4) Determine whether the vector $\vec{x}$ is in $V = \text{span}(\vec{v}_1, \ldots, \vec{v}_m)$. If $\vec{x} \in V$, find the coordinates of $\vec{x}$ with respect to the basis $\mathfrak{B} = (\vec{v}_1, \ldots, \vec{v}_m)$ of V, and write the coordinate vector $[\vec{x}]_{\mathfrak{B}}$:

$$\vec{x} = \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}; \quad \vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

0xA. (§3.4) Find the matrix B of the linear transformation $T(\vec{x}) = A\vec{x}$, with respect to the basis $\mathfrak{B} = (\vec{v}_1, \ldots, \vec{v}_m)$.

$$A = \begin{bmatrix} 4 & 2 & -4 \\ 2 & 1 & -2 \\ -4 & {=}\,2 & 4 \end{bmatrix}; \quad \vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

0xB. (§2.2, §3.1, §3.2, §3.3) Describe *(i)* the *image*, $\text{im}(A)$; and *(ii)* the *kernel*, $\ker(A)$ of the transformation $T(\vec{x}) = A\vec{x}$ geometrically, where

$$T(\vec{x}) = \left\{ \begin{array}{c} \text{ORTHOGONAL PROJECTION ONTO THE PLANE} \\ \{x + 2y + 3z = 0\} \text{ IN } \mathbb{R}^3 \end{array} \right\}.$$

what is *(iii)* $\dim(\text{im}(A))$, and *(iv)* $\dim(\ker(A))$? Then find a basis for *(v)* $\text{im}(A)$ and *(vi)* $\ker(A)$