1	2	3	4	5	6	7	8	9
RedID								

Math 254, Spring 2017
Midterm #3 Practice Test

Tools: Brain/Pen/Pencil/Eraser/Paper.
Rules: This is just a test-test; see below:

v 2017.4.18.0

I, ____________________________, understand that this is a practice exam; which is provided for adults and entertainment purposes only. Any resemblence to a real test is purely intentional, but not guaranteed.

____
Signature (REQUIRED to continue (yeah, that will be enforced?!?))

Suggestions:

- Present your solutions using standard notation in an easy-to-read format. It is your job to yourself you did the problem correctly...
- ***Your answers MUST logically follow from your calculations in order to be considered! ("Miracle solutions" ⇒ zero points.)***
- If you are a computer science major, perform all calculations in the hexadecimal basis.

Note:

- There is a high probablility that the actual test will have slightly fewer questions.

Problem	Pts Possible	Pts Scored
1	000	
2	000	
3	000	
4	000	
5	000	
6	000	
8	000	
9	000	
0xA	000	
0xB	000	
0xC	000	
Total	000	

1. (§3.1–§5.3, The Mother of All Problems!) Consider the matrix, $A \in \mathbb{R}^{r \times c}$:

$$A = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & -1 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

and the linear transformation $T(\vec{x}) = A\vec{x}$.

(a) What can you say about the relations between

i. $\dim(\operatorname{im}(A))$, and $\dim(\ker(A))$

ii. $\dim(\operatorname{im}(A))$, and $\dim(\operatorname{im}(A)^{\perp})$

iii. $\dim(\ker(A))$, and $\dim(\ker(A)^{\perp})$

The following sketch may be useful (or useless):

$$\begin{bmatrix} \mathbb{R}^m \\ \\ \ker(A)^{\perp} \\ \ker(A) \end{bmatrix} \overset{T(\vec{x})}{\longrightarrow} \begin{bmatrix} \mathbb{R}^n \\ \operatorname{im}(A)^{\perp} \\ \operatorname{im}(A) \\ \vec{0} \end{bmatrix}$$

(b) Find Bases for

i. $\operatorname{im}(A)$.

ii. $\ker(A)$.

iii. $\operatorname{im}(A)^{\perp}$.

iv. $\ker(A)^{\perp}$.

(c) Find Orthonormal Bases for (and forever curse Jørgen Pedersen Gram and Erhard Schmidt)

i. $\operatorname{im}(A)$.

ii. $\ker(A)$.

iii. $\operatorname{im}(A)^{\perp}$.

iv. $\ker(A)^{\perp}$.

(d) Use the QR-factorization to relate the answers {(b).i–iv} and {(c).i–iv}.

2. (§5.1) Find the orthogonal projections of $\vec{e}_1$ onto the subspaces W, and $W^{\perp}$ of $\mathbb{R}^4$, where

$$W = \operatorname{span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \right), \quad \vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

3. (§5.2) Perform the Gram-Schmidt process on the sequence of vectors given:

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 18 \\ 0 \\ 0 \end{bmatrix}.$$

4. (§5.2) Find an orthonormal basis for the image of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & -2 & 0 \end{bmatrix}.$$

5. (§5.3) Are the given matrices Orthogonal?

$$A = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & 0.6 \end{bmatrix}, \quad B = \begin{bmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix}$$

6. (§5.3)

   (a) Consider an $n \times m$ matrix A such that $A^T A = I_m$. Is is necessarily true that $AA^T = I_n$? (Explain!)

   (b) Consider an $n \times n$ matrix A such that $A^T A = I_n$. Is is necessarily true that $AA^T = I_n$? (Explain!)

7. (§6.1) Use the determinant to decide whether the matrices are invertible:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 5 & 7 \\ 0 & 11 & 7 \\ 0 & 0 & 5 \end{bmatrix}$$

8. (§6.1) For which values of k, ℓ are the matrices invertible?

$$A = \begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} \ell & 3 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 4 \end{bmatrix}$$

9. (§6.1–§6.2) Given the value of $\det(A)$, what is the value of

   (a) $\det(A^T)$

   (b) $\det(A^{-1})$

   (c) $\det(A^2)$

   (d) $\det(A^T A^{-1})$

0xA. (§6.2) Ponder the matrix

$$A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 4 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \end{bmatrix}$$

compute the determinant using

   (a) The Combinatorial "Pattern" Method

   (b) Row Reductions (Gaussian Elimination)

   (c) The Laplace co-factor Method

0xB. (§6.2) Given the values of $\det(A)$ and $\det(B)$ is the value of $\det(AB)$ always defined? (Explain) If/when it is defined, what is it?

0xC. (§6.2) Explain how the Row Reduction operations impact the value of the determinant:

   (a) Row swap

   (b) Row scaling

   (c) Row addition

   (d) In the light of 9.(a) what does (a)–(c) imply for column operations?