Math 524: Linear Algebra: A Second Course

Notes #0 — #include ⟨hello_world⟩

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Fall 2021

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Outline

San Diego State University

Academic Life

MSc

- KTH MSc. Engineering Physics, Royal Institute of Technology (KTH), Stockholm, Sweden. Thesis Advisers: Michael Benedicks, Department of Mathematics KTH, and Erik Aurell, Stockholm University, Department of Mathematics. Thesis Topic: *"A Renormalization Technique for Families with Flat Maxima."*

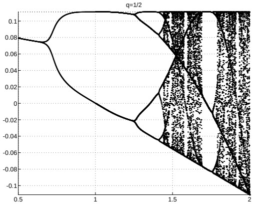

Figure: Bifurcation diagram for the family $f_{a,\frac{1}{2}}$ [Blomgren-1994]

Academic Life — PhD

- UCLA PhD. UCLA Department of Mathematics. Adviser: Tony F. Chan. PDE-Based Methods for Image Processing. Thesis title: *"Total Variation Methods for Restoration of Vector Valued Images."*

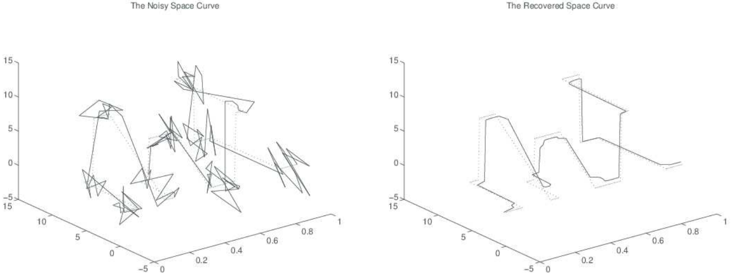

Figure: The noisy (SNR = 4.62 dB), and recovered space curves. Notice how the edges are recovered. [Blomgren-1998]

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Academic Life — Postdoc

- Research Associate. Stanford University, Department of Mathematics. Main Focus: Time Reversal and Imaging in Random Media (with George Papanicolaou, *et. al.*)

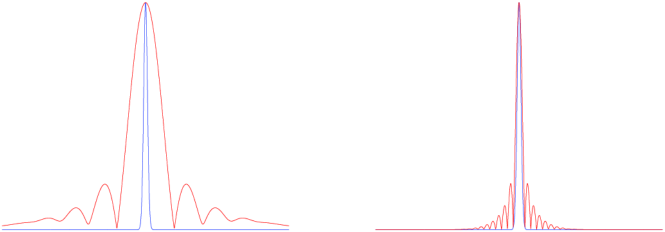


Figure: Comparison of the theoretical formula for a medium with $L = 600\,m$, $a_e = 195\,m$, $\gamma = 2.12 \times 10^{-5}\,m^{-1}$. [LEFT] shows a homogeneous medium, $\gamma = 0$, with $a = 40\,m$ TRM (in red / wide Fresnel zone), and a random medium with $\gamma = 2.12 \times 10^{-5}$ (in blue). [RIGHT] shows $\gamma = 0$, with $a = a_e = 195\,m$ (in red), and $\gamma = 2.12 \times 10^{-5}$, with $a = 40\,m$ (in blue). The match confirms the validity of [the theory]. [BLOMGREN-PAPANICOLAOU-ZHAO-2002]

Academic Life

- San Diego State University Professor, SDSU, Department of Mathematics and Statistics. Projects: Computational Combustion, Biomedical Imaging (Mitochondrial Structures, Heartcell Contractility, Skin/Prostate Cancer Classification), carbon sequestration, compressed sensing.

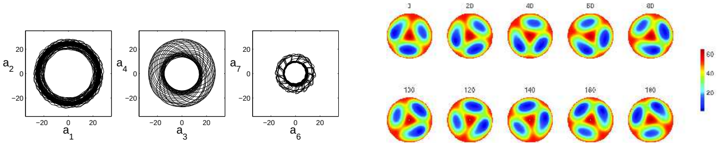


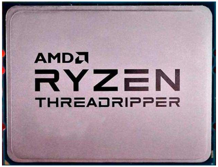
Figure: [Left] Phase-space projections produced by the time coefficients of the POD decomposition of the rotating pattern shown in [Right]. [Blomgren-Gasner-Palacios-2005]

Primary Research Interests — Current — High Performance Computing

Development of algorithms achieving near-optimal GPU utilization, with applications to Computational PDEs, Computational Linear Algebra, and Computational Optimization.

Project #1: Fast Multipole Method for *Waves over Vortices*, w/Chris Curtis & Daniel Matteson. (S'21)

Project #2: ???, w/*???* & ???



3990x, ≈ $5,500
64 Cores, 128 Threads.

RTX 3090, ≈ $1,499*
10,496 CUDA Cores

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Fun Times... ⇔ Endurance Sports

- Triathlons:
  - (13) Ironman distance $(2.4 + 112 + 26.2)$ — [PR] 11:48:57
  - (16) Half Ironman distance — 5:14:20
- Running
  - (1) 100k Race (62.1 miles) — 15:37:46 (15:05/mi)
  - (1) Trail Double-marathon (52 miles) — 10:59:00 (12:32/mi)
  - (5) Trail 50-mile races — 9:08:46 (10:59/mi)
  - (8) Trail 50k (31 mile) races — 5:20:57 (10:20/mi)
  - (16) Road/Trail Marathons — 3:26:19 (7:52/mi)
  - (30) Road/Trail Half Marathons — 1:35:00 (7:15/mi)

Contact Information

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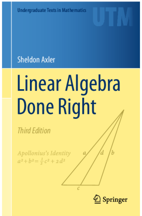
Office Hours	**https://calendly.com/blomgren_sdsu/**

https://Canvas.SDSU.edu/ is the *de facto* syllabus for the class.

The Book

Our main reference is the same book used in **X Development LLC**'s[‡] internal linear algebra course:

- *Linear Algebra Done Right* (3rd edition), Sheldon Axler, Springer International Publishing 2015.
  - ISBN 978-3-319-11079-0
  - DOI 10.1007/978-3-319-11080-6
  - ISSN 2197-5604 (electronic)
  - ISBN 978-3-319-11080-6 (eBook)



‡ Formerly known as "`Google X`." (Google's semi-secret R&D subsidiary) — The Google seach-engine is one of the most famous applications of the Singular Value Decomposition (SVD), $A = U\Sigma V^*$.

Covid-19 Hardened Grading Scheme

- **Three (3) In-Person Tests ⇐ "Planned"**

  55% Homework
  10% Midterm 1 (Take-Home)
  5% Midterm 1 (In-Class)
  10% Midterm 2 (Take-Home)
  5% Midterm 2 (In-Class)
  10% Final (Take-Home)
  5% Final (In-Class)

- Two (2) In-Person Tests

  60% Homework
  10% Midterm 1 (Take-Home)
  10% Midterm 2 (Take-Home)
  10% Final (Take-Home)
  5% In-person Test 1
  5% In-person Test 2

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Covid-19 Hardened Grading Scheme

- One (1) In-Person Test

65%	Homework
10%	Midterm 1 (Take-Home)
10%	Midterm 2 (Take-Home)
10%	Final (Take-Home)
5%	In-person Test 1

- No (0) In-Person Tests

70%	Homework
10%	Midterm 1 (Take-Home)
10%	Midterm 2 (Take-Home)
10%	Final (Take-Home)

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Prerequisites

"Required:" Mathematics 245, and either 254 or 342A (alt AE-280) with a grade of C (2.0) or better in each course.

Highly recommended: at least one 300-level theoretical/formal Mathematics course, *e.g.* 320, 330; or 300-level class(es) in Engineering/Computer Science/Physics.

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Expectations and Procedures, I

> Lecture recordings will be posted (linked in Canvas);
> no-questions-asked unlimited absence policy for lectures.

- Class attendance is (α) HIGHLY RECOMMENDED — Homework and announcements will be posted on the class web page; or (β) **MANDATORY** for ALL in-class tests/presentations. If/when you attend class:
  - Please be on time.
  - Please pay attention.
  - Please turn mobile phones to "silent."
  - Please be courteous to other students and the instructor.
  - Abide by university statutes, and all applicable local, state, and federal laws.

Expectations and Procedures, II

- Please, turn in assignments on time. (The instructor reserves the right not to accept late assignments.)
- The instructor will make special arrangements for students with documented learning disabilities and will `try` to make accommodations for other unforeseen circumstances, *e.g.* illness, personal/family crises, etc. in a way that is fair to all students enrolled in the class. **Please contact the instructor EARLY regarding special circumstances.**
- Students are expected **and encouraged** to ask questions in class!
- Students are expected **and encouraged** to to make use of office hours! If you cannot make it to the scheduled office hours: contact the instructor to schedule an appointment!

Expectations and Procedures, II$\frac{I}{II}$

Late HW Policy

- Assignments accepted up to 24 hours after original deadline, with a 10% penalty.
- Further extensions will only be granted in extreme, well-documented, circumstances.

Expectations and Procedures, III

Three (3) in-person tests planned for Fall 2021.
Possibility: SDSU Testing Center (fee applies), if you are not comfortable taking the test in a high-capacity classroom.

- Missed midterm exams: Don't miss exams! The instructor reserves the right to schedule make-up exams, make such exams oral presentation, and/or base the grade solely on other work (including the final exam).
- Missed final exam/presentation: Don't miss the final! Contact the instructor ASAP or a grade of `WU` or `F` will be assigned.
- **Academic honesty**: submit your own work — but feel free to discuss homework with other students in the class! It's OK to ask "Uncle Google" and "Aunt Wiki" for help and ideas, but process the information and make it your own, AND cite any and all sources (outside of class material) you use.

Honesty Pledges, I

- The following **Honesty Pledge** must be included in all programs you submit (as part of homework and/or projects):
  - I, (your name), pledge that this program is completely my own work, and that I did not take, borrow or steal code from any other person, and that I did not allow any other person to use, have, borrow or steal portions of my code. I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies.
- Work missing the honesty pledge **may not be graded!**

Honesty Pledges, II

- Larger reports must contain the following text:
  - I, (your name), pledge that this report is completely my own work, and that I did not take, borrow or steal any portions from any other person. Any and all references I used are clearly cited in the text. I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies. `Your signature`.
- Work missing the honesty pledge `may not be graded!`

Why?

There is a LOT of Linear Algebra beyond the basic Matrix-Vector material covered in Math 254.

What??

Multiple goals:

Local

- Provide the "theoretical glue" between Math 254 (Intro to Linear Algebra) and Math 543 (Computational Linear Algebra)
- Note: Math 524 is NOT a prerequisite for Math 543
- Make 254–524–543 a logically coherent collection of classes.

Global Develop "mathematical maturity" — abstraction, theory, and proofs.

- Deeper understanding of linear algebra.
- Help prepare students for graduate-level mathematics

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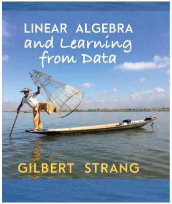
How???

- We will follow the notation and structure of Axler's *Linear Algebra Done Right*, and add relevant connections to various applications, etc...

- As appropriate, we'll pull in topics / applications from Gil Strang's book *Linear Algebra and Learning from Data* (2019).



- This is mainly a theoretical *math* class, but we keep applications in mind.

Class Goals (Proto-"Learning Objectives")

The big picture Class Goals are:

- *Deep* understanding and appreciation of:
  - Vector spaces; Linear maps and their associated matrices; subspace theory; eigen-values – eigen-vectors – invariant subspaces; inner product spaces and operators thereon.
- Mastery of Proof Techniques:
  - Direct Proofs, Proofs by { induction, contraposition, contradiction, construction }, Non-constructive proofs

  Demonstrated by proof-writing and validation.
- The above will have a profound impact on *Matrix Factorizations*:
  - Ability to identify the right type of matrix factorization for a particular problem situation / application.
  - Ability to perform, and apply an appropriate matrix factorization to solve a problem / advance the solution in an applied problem.

Each lecture will define 3 – 5 **actionable** **Student Learning Objectives**, to help us navigate the material; these constitute the "contract" of what is important in the class.