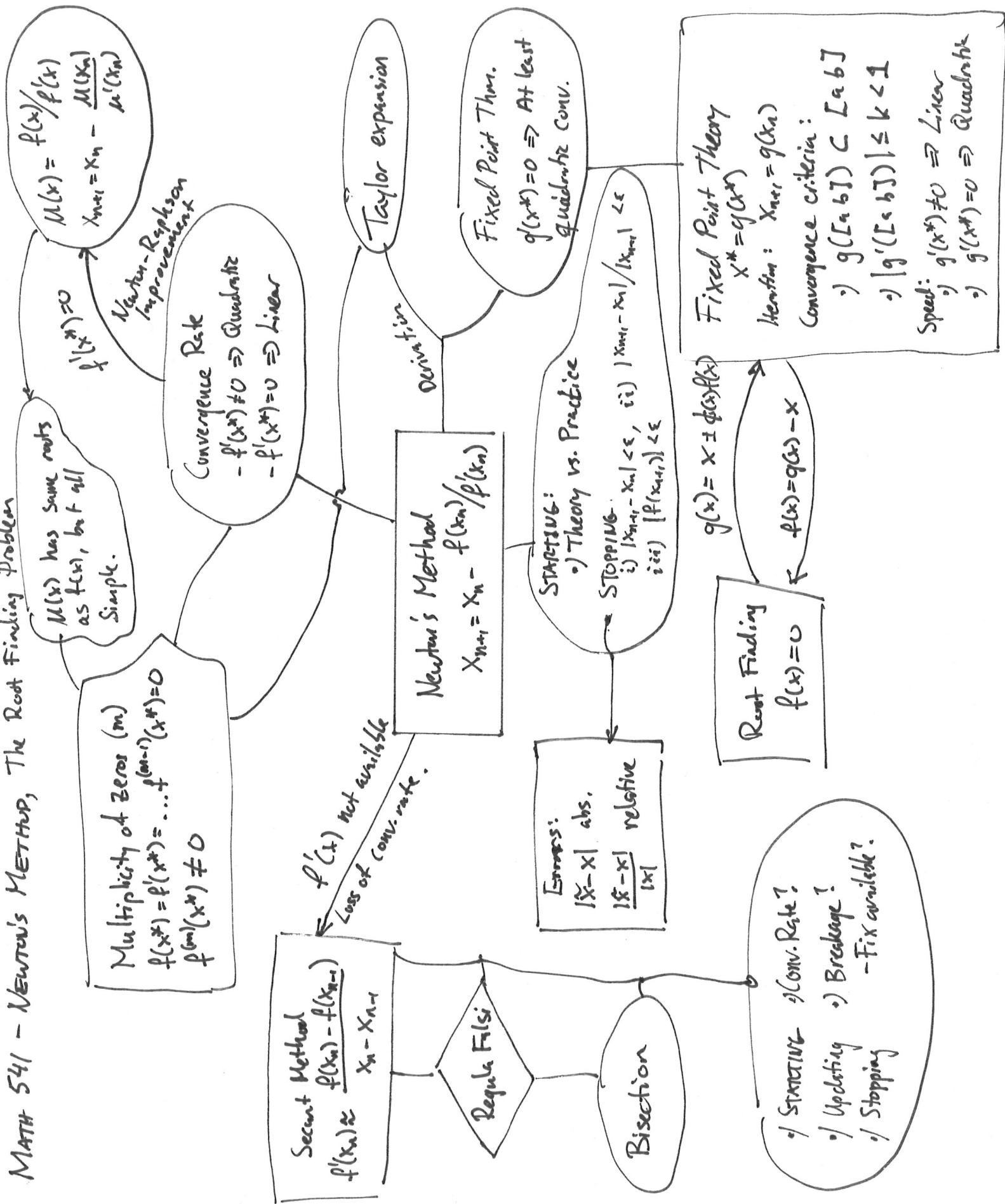


# MATH 541 - NEWTON'S METHOD, THE ROOT FINDING PROBLEM



$M(x)$  has same roots as  $f(x)$ , but all Simple.

Convergence Rate  
-  $f'(x^*) \neq 0 \Rightarrow$  Quadratic  
-  $f'(x^*) = 0 \Rightarrow$  Linear

Multiplicity of zeros (m)  
 $f(x^*) = f'(x^*) = \dots = f^{(m-1)}(x^*) = 0$   
 $f^{(m)}(x^*) \neq 0$

$f'(x)$  not available  
Loss of conv. rate.

Errors:  
 $|x - x^*|$  abs.  
 $\frac{|x - x^*|}{|x|}$  relative

Root Finding  
 $f(x) = 0$   
 $g(x) = x - f(x)$

Stopping criteria:  
- Convergence Rate?  
- Breakage?  
- Fix available?

Fixed Point Theory  
 $x^* = g(x^*)$   
Iteration:  $x_{n+1} = g(x_n)$   
Convergence criteria:  
-  $g([a, b]) \subset [a, b]$   
-  $|g'([c, d])| \leq k < 1$   
Speed:  
-  $g'(x^*) \neq 0 \Rightarrow$  Linear  
-  $g'(x^*) = 0 \Rightarrow$  Quadratic

Newton's Method  
 $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

STARTING:  
- Theory vs. Practice  
STOPPING:  
i)  $|x_{n+1} - x_n| \leq \epsilon$ , ii)  $|x_{n+1} - x_n| / |x_n| \leq \epsilon$   
iii)  $|f(x_{n+1})| \leq \epsilon$

Derivatives  
Taylor expansion  
Fixed Point Thm.  
 $g(x^*) = 0 \Rightarrow$  At least Quadratic Conv.

Newton-Raphson Improvement  
 $f'(x^*) = 0$   
 $M(x) = \frac{f(x)}{f'(x)}$   
 $x_{n+1} = x_n - \frac{M(x_n)}{M'(x_n)}$