Math 542 Spring 2015 — Assignment #5. Due 4/10/2015.

The n-Body Problem

The n-body problem is the problem of finding, given the initial positions, masses, and velocities of n bodies, their subsequent motions as determined by classical mechanics, i.e., Newton's laws of motion and Newton's law of gravity.

For given initial data $\vec{x}_k$, $k = 1, \ldots, n$ we get the equations

$$m_j \frac{d^2}{dt^2}\vec{x}_j = \gamma \sum_{k \neq j} \frac{m_j m_k (\vec{x}_k - \vec{x}_j)}{\|\vec{x}_k - \vec{x}_j\|^3}, \quad j = 1, \ldots, n$$

where m_j are the masses, and $\vec{x}_j = (x_j, y_j, z_j)$ are the positions of the objects. γ is the gravitational constant.

The $(n = 2)$-body problem was completely solved by Johann Bernoulli (1667–1748). But for $n \geq 3$ no "easy" analytical solutions exist (in general). Even the $(n = 3)$-body problem can give rise to chaotic solutions.

Your task is to simulate the three-body system corresponding to the Sun–Earth–Moon system, using a numerical scheme (of your choice) of at least order 4. Run the simulation for a time-interval corresponding to at least 5 years (and make sure no cosmic disasters occur!)

Talk about any "engineering" decisions you made in your code in terms of normalizing parameters, step-lengths, and on-line or off-line error analysis.

Note that this problem is sort of vague and open-ended, and this description does not include all the information you need to solve the problem. Just like the real-world. :-)

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[0]Resources:

http://www.nasa.gov/
http://www.jpl.nasa.gov/
http://en.wikipedia.org/wiki/Sun
http://en.wikipedia.org/wiki/Earth
http://en.wikipedia.org/wiki/Earth's_orbit
http://en.wikipedia.org/wiki/Moon
http://en.wikipedia.org/wiki/Orbit_of_the_Moon