Numerical Matrix Analysis

Notes #1 — Introduction

Peter Blomgren
⟨blomgren@sdsu.edu⟩

Department of Mathematics and Statistics
Dynamical Systems Group
Computational Sciences Research Center
San Diego State University
San Diego, CA 92182-7720

http://terminus.sdsu.edu/

Spring 2026

(Revised: January 26, 2026)

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Outline

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   - Non-Academic Life
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   - Expectations and Procedures
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   - Formal Prerequisites
4. Introduction
   - The "Why?" the "What?" and the "How?"

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Academic Life

MSc

- MSc. Engineering Physics, Royal Institute of Technology (KTH), Stockholm, Sweden. Thesis Advisers: Michael Benedicks, Department of Mathematics KTH, and Erik Aurell, Stockholm University, Department of Mathematics. Thesis Topic: *"A Renormalization Technique for Families with Flat Maxima."*

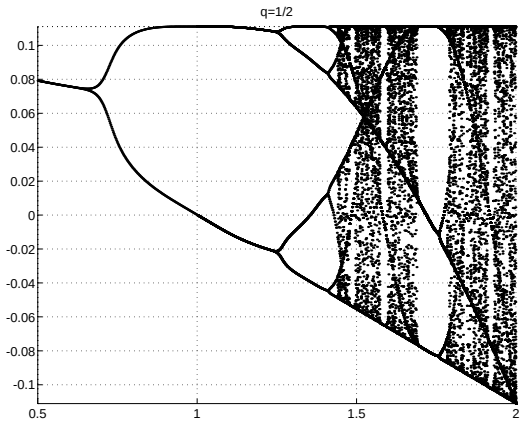

Figure: Bifurcation diagram for the family $f_{a,\frac{1}{2}}$ [Blomgren-1994]

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Academic Life

PhD

- UCLA PhD. UCLA Department of Mathematics. Adviser: Tony F. Chan. PDE-Based Methods for Image Processing. Thesis title: *"Total Variation Methods for Restoration of Vector Valued Images."*

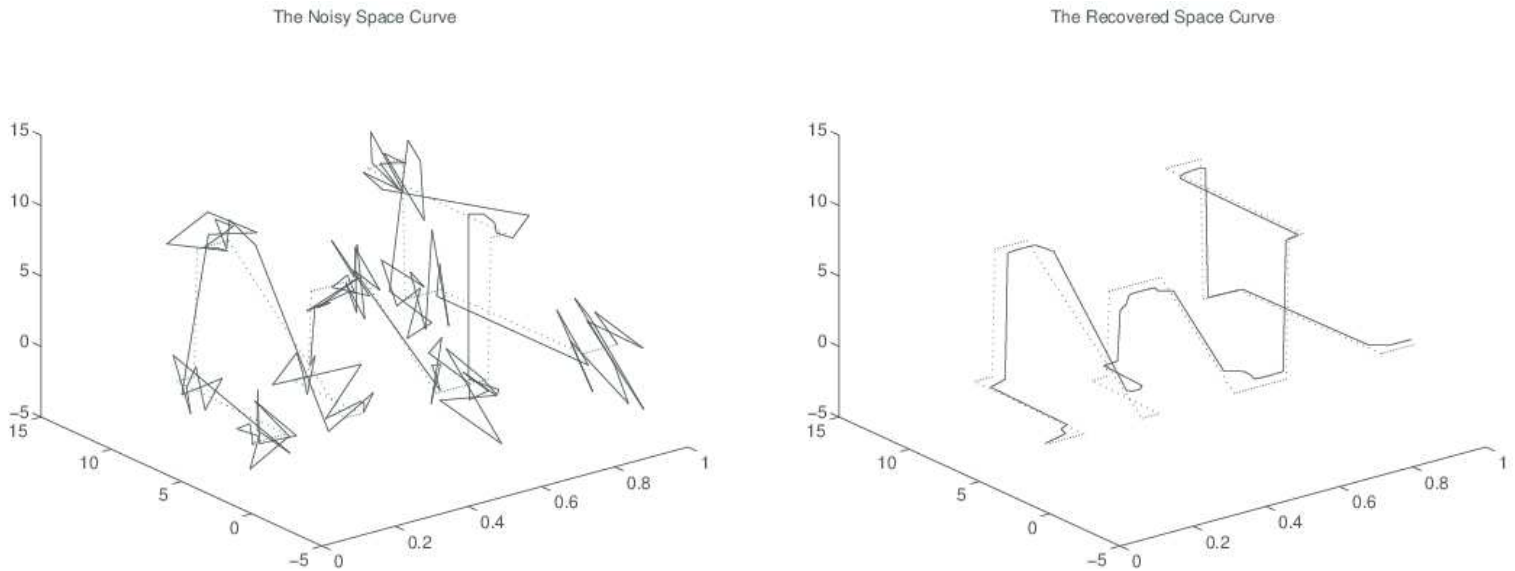

Figure: The noisy (SNR = 4.62 dB), and recovered space curves. Notice how the edges are recovered. [BLOMGREN-1998]

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Academic Life

Postdoc

-  Research Associate. Stanford University, Department of Mathematics. Main Focus: Time Reversal and Imaging in Random Media (with George Papanicolaou, *et. al.*)

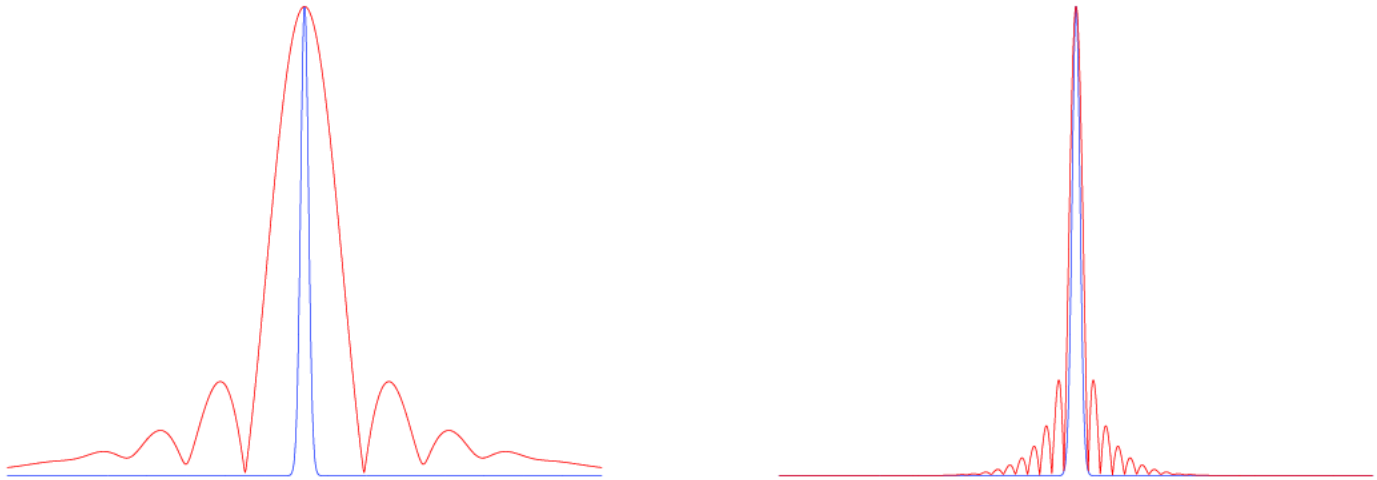

Figure: Comparison of the theoretical formula for a medium with $L = 600\,m$, $a_e = 195\,m$, $\gamma = 2.12 \times 10^{-5}\,m^{-1}$. [LEFT] shows a homogeneous medium, $\gamma = 0$, with $a = 40\,m$ TRM (in red / wide Fresnel zone), and a random medium with $\gamma = 2.12 \times 10^{-5}$ (in blue). [RIGHT] shows $\gamma = 0$, with $a = a_e = 195\,m$ (in red), and $\gamma = 2.12 \times 10^{-5}$, with $a = 40\,m$ (in blue). The match confirms the validity of [the theory]. [BLOMGREN-PAPANICOLAOU-ZHAO-2002]

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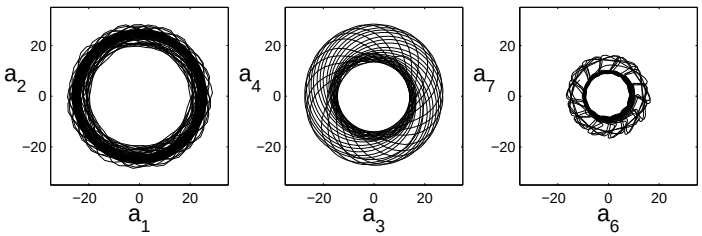
Academic Life

Professor



- SAN DIEGO STATE UNIVERSITY Professor, SDSU, Department of Mathematics and Statistics. Projects: Computational Combustion, Biomedical Imaging (Mitochondrial Structures, Heartcell Contractility, Skin/Prostate Cancer Classification), carbon sequestration, compressed sensing.



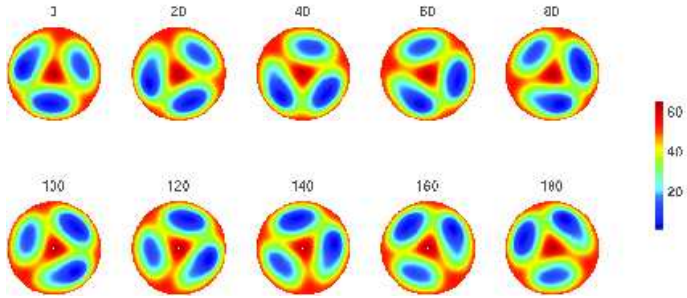


Figure: [LEFT] Phase-space projections produced by the time coefficients of the POD decomposition of the rotating pattern shown in [RIGHT]. [BLOMGREN-GASNER-PALACIOS-2005]



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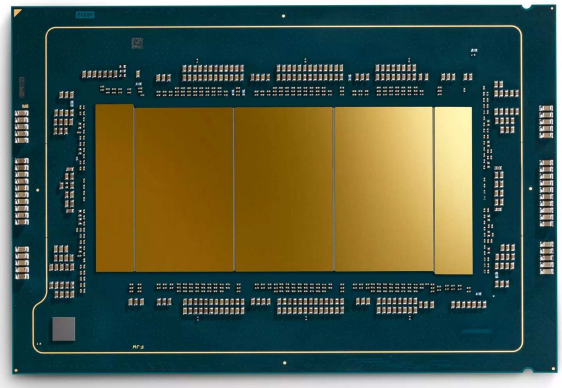
Primary Research Interests — Current — High Performance Computing, DS/AI

Development of algorithms achieving near-optimal GPU utilization, with applications to Computational PDEs, Computational Linear Algebra, and Computational Optimization.

#1: *Fast Multipole Method for Waves over Vortices*, w/Chris Curtis & Daniel Matteson. (S'21)

#2: *StarPU + ScalFMM Implementation for the Fast Multipole Method*, w/___. (abandoned)

Current "Top of the Line" Hardware — Commercially "Available" CPU / GPU



Intel® Xeon® 6980P Processor
MSRP $12,460
128 CPU Cores, 256 Threads.



RTX 5090
MSRP $1,999
21,760 CUDA Cores,
32GB GDDR7

RTX PRO 6000
MSRP $8,600
24,064 CUDA Cores,
96GB GDDR7

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Fun Times... $\Leftrightarrow$ Endurance Sports

Pre-Pandemic...



- Triathlons:
  - (13) Ironman distance (2.4 + 112 + 26.2) [PR] 11:48:57
  - (16) Half Ironman distance 5:14:20
- Running
  - (1) 100k Race (62.1 miles) 15:37:46 (15:05/mi)
  - (1) Trail Double-marathon (52 miles) 10:59:00 (12:32/mi)
  - (5) Trail 50-mile races 9:08:46 (10:59/mi)
  - (8) Trail 50k (31 mile) races 5:20:57 (10:20/mi)
  - (16) Road/Trail Marathons 3:26:19 (7:52/mi)
  - (30) Road/Trail Half Marathons 1:35:00 (7:15/mi)

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Contact Information



Office	GMCS-415 and zoom - zoom - zoom
Email	**blomgren@sdsu.edu**
Web	**https://canvas.sdsu.edu/** **https://www.gradescope.com** **http://terminus.sdsu.edu/SDSU/Math543**
Office Hours	Catch Peter in GMCS-415C Highest probability Tu/Th mornings before class or send an email to set up an appointment

* Generally, I am planning on being on-campus 10am – 5pm on Tu/Th in Spring 2026.

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The Post(?) Pandemic Reality

- S-2020: We went online part-way thru the semester
- S-2021: Zoom-U
- S-2022: Masked-U — not a good semester (many reasons)
- S-2023: The New Normal^{TM}... some modifications.
- S-2024: The "AI"-era arrived + 1-day CFA (1-day) strike.
- S-2025: "AI" is quite a bit more "mature" than last year...
- S-2026: Again, "AI" is quite a bit more "mature" than last year...

During COVID we all de-socialized to some extent; and there is definitely a "learning gap."

There is sometimes a tendency to see all this silly learning and class work as unnecessary obstacles to getting a degree.

... which leads to the temptation of taking the path of least resistance and have Uncle Google, Aunt Wiki, Scuzzy Cousin Chegg, Skynet, or ChatGPT/Gemeni/Claude/... do all the heavy lifting... in a lazy way.



The Post$^{(?)}$ Pandemic Reality

- We need to remind ourselves that education is about *developing skills and processes*, definitely not just about "having The Answer." Understanding
  - how to get The Answer;
  - how to validate The Answer;
  - what The Answer means; and
  - how The Answer potentially is useful;

  are often the bigger and more important lessons.
- It is "unlikely" that you, in real life, will be asked to differentiate $\cos(\sin(\tan(\ln(x))))$, or write the 55,000,001st analysis of Fyodor Dostoevsky's *"Crime and Punishment,"* or single-handedly implement the Singular Value Decomposition... but acquiring the skills to perform these tasks are arguably useful.

The Post(?) Pandemic Reality

New AI-era Policies — SPRING 2024, revised 2025

AI-3 Documented: *Students can use AI in any manner for this assessment or deliverable, but they* **must provide appropriate documentation for all AI use.**

This applies to ALL MATH-543 WORK during the SPRING 2026 semester.

The goal is to leverage existing tools and resources to generate HIGH QUALITY SOLUTIONS to all assessments.

You MUST document what tools you use and HOW they were used (including prompts); AND how results were VALIDATED. **A significant portion (10-20%) of the grade/score will be associated with discussing tools/references used.**

BE PREPARED to DISCUSS homework solutions and AI-strategies. **Participation in the in-class discussions will be an essential component of the grade for each assessment.**



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The Post(?) Pandemic Reality

Accessibility Statement (Spring 2026)
The lecture slides are created using the `beamer` class of the LaTeX markup "language". LaTeX-`beamer` does NOT produce ADA-compliant `WTPDF/PDF/UA-2` or `PDF/UA-1` accessible documents. A new law takes effect April 24 2026, requiring that ALL electronically distributed materials be compliant. There is an ongoing effort to update the LaTeX code base to produce property tagged accessible PDFs. The `beamer` class is being replaced by the compliant `ltx-talk` class; however, I have somewhere in the range of 8,000 - 10,000 slides to convert which may take "a while." While the migration is "ongoing," non-compliant course slides CANNOT be distributed(*). Instead each lecture will be supplemented by screen-reader friendly companion documents. The companion documents will be generated/updated during the semester and posted withing 24 hours of each lecture.

(*) "Archived Materials" distributed in prior semesters can remain public facing; e.g. you can find the Spring 2025 slid at:
`http://terminus.sdsu.edu/SDSU/Math543_s2025/?r=sched`

Note: The Math 543 canvas shell is currently 100% compliant as measured by the UDOIT Course Accessibility.

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New Lecture for Spring 2026

Working Number: Lecture #1.5
Working Title: *"Can I Have Your Attention?!?"*

Discussing the role of Computational Linear Algebra during Era of the First Rise of our AI Overlords.

"Attention" is of course part of the AI/Machine Learning process; as described in *"Attention Is All You Need"*, Vaswani, et.al., Advances in neural information processing systems, volume 30, 2017.

`https://proceedings.neurips.cc/paper/2017/file/3f5ee243547dee91fbd053c1c4a845aa-Paper.pdf`

This lecture will be given as soon as it is "done." (Ideal Target: Thursday 22 January; more likely Tuesday 27 or Thursday 29 January).



New Homework for Spring 2026

Fair Warning:
As currently mapped there are 8 assignments $\{1, 2, 3, 4, 5, 6, 6.5,$ and $7\}$; I am planning on adding one additional assignment in the latter half of the semester, and maybe re-map to $\{1, 2, 3, 4, 5, 6, 7, 8,$ and $9\}$.

Remodeling in Process:
Reframing the context and applications to align with AI/ML applications, without abandoning "classical" applied/computational math applications.

Diversity and Non-Discrimination Statement

Diversity and Non-Discrimination
This course values an inclusive learning environment and does not tolerate discrimination, harassment, or retaliation. All students are expected to engage respectfully with others and with course content. If you experience or observe bias, harassment, or discrimination, please contact me and/or report the concern to Dr. Estralita Martin (Assistant Dean for Student Affairs, Director of CASA).

Names and Pronouns: I will gladly use your stated name and pronouns. Please let me know your preferences (in person, by email, or during office hours), and I will do my best to honor them consistently (just be aware the I'm suffering from early-stage CRS (Can't Remember S...tuff!)).

Access and Inclusion: If you have a disability, health condition, or other access need that may affect your participation, please connect with SDSU Student Disability Services and let me know how I can support your learning within the course structure.

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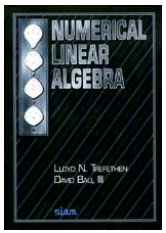
Math 543: Literature

"Required" —



Numerical Linear Algebra, Lloyd N. Trefethen and David Bau, III, Society for Industrial and Applied Mathematics (SIAM*), 1997. ISBN 978-0-898713-61-9.

Also: Twenty-fifth Anniversary Edition, Society for Industrial and Applied Mathematics (SIAM), 2022. ISBN: 978-1-611977-15-8

"Required" — (Supplemental)

Class notes and class canvas-page.

* SIAM members receive special pricing (30% off). SIAM student membership is free.

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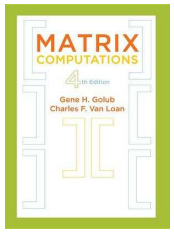
Math 543: Literature

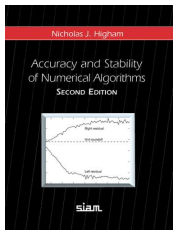
Everything You Ever Wanted to Know, but Were Afraid to Ask...

"Optional" — (Classic, Comprehensive Reference)



Matrix Computations, 4th Edition, Gene H. Golub and Charles F. Van Loan, Johns Hopkins University Press, 2012. ISBN-13 978-1421407944.

"Optional" — (Comprehensive Reference)



Accuracy and Stability of Numerical Algorithms, Second Edition, Nicholas J. Higham, Society for Industrial and Applied Mathematics (SIAM*), 2002. ISBN 978-0-898715-21-7.

* SIAM members receive special pricing (30% off). SIAM student membership is free.

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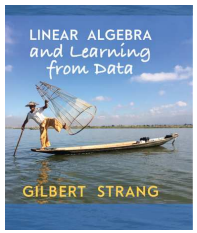
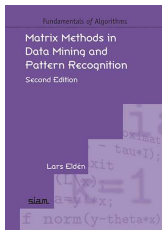
Math 543: Literature

"Inspiration for the Road Ahead" —

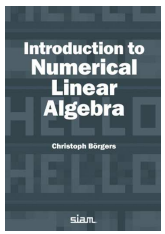


Linear Algebra and Learning From Data, Gilbert Strang, Society for Industrial and Applied Mathematics (SIAM*), 2019. ISBN 978-069219638-0.

"Considered... but, no." —



Matrix Methods in Data Mining and Pattern Recognition, Second Edition, Lars Eldén, Society for Industrial and Applied Mathematics (SIAM*), 2019. ISBN: 978-1-611975-85-7.



Introduction to Numerical Linear Algebra, Christoph Börgers, Society for Industrial and Applied Mathematics (SIAM*), 2022. ISBN: 978-1-61197-691-5.

* SIAM members receive special pricing (30% off). SIAM student membership is free.

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Math 543: Introduction — Grading etc.

50% `Homework:` both theoretical, and implementation (programming) — Recommended languages: Python, Matlab, C/C++, or Fortran; however anything goes: 6510 assembler, Java, M$-D$^\flat$, Haskell...

25% `Midterm:` [$\frac{1}{2}$ Take-Home, and $\frac{1}{2}$ ~~In-Class~~].

25% `Final:` [$\frac{1}{2}$ Take-Home, and $\frac{1}{2}$ ~~In-Class~~].

Expectations and Procedures, I

- Most class attendance is "OPTIONAL" — Homework and announcements will be posted on the class canvas page. If/when you attend class:
  - Please be on time.
  - Please pay attention.
  - Please turn "off" mobile phones.



  - Please be courteous to other students and the instructor.
  - Abide by university statutes, and all applicable local, state, and federal laws.
  - **Spring 2026**: Note that participation in in-class discussions of assignments is REQUIRED. Discussions will take place approximately 1 week after the deadline for each assignment.

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Expectations and Procedures, II

- Please, turn in assignments on time. (The instructor reserves the right not to accept late assignments.)
- The instructor will make special arrangements for students with documented learning disabilities and will `try` to make accommodations for other unforeseen circumstances, *e.g.* illness, personal/family crises, etc. in a way that is fair to all students enrolled in the class. **Please contact the instructor EARLY regarding special circumstances.**
- Students are expected **and encouraged** to ask questions in class!
- Students are expected **and encouraged** to to make use of office hours! If you cannot make it to the scheduled office hours: contact the instructor to schedule an appointment!



Expectations and Procedures, II/II

Updated Spring 2022

Late HW Policy

- Assignments accepted up to 24 hours after original deadline, with a 10% penalty.
- Further extensions will only be granted in extreme, well-documented, circumstances.

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Expectations and Procedures, III

- Missed midterm exams: Don't miss exams! The instructor reserves the right to schedule make-up exams, make such exams oral presentation, and/or base the grade solely on other work (including the final exam).
- Missed final exam: Don't miss the final! Contact the instructor ASAP or a grade of incomplete or F will be assigned.
- **Academic honesty**: submit your own$^{(*)}$ work — but feel free to discuss homework with other students in the class!

$^{(*)}$ see also AI-policies for the class.

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Honesty Pledges, I

- The following **Honesty Pledge** must be included in all programs you submit (as part of homework and/or projects):
  - I, `[your name]`, pledge that this program is completely my own work, and that I did not take, borrow or steal code from any other person — real or artificial — and that I did not allow any other person to use, have, borrow or steal portions of my code. **I have adhered to the AI-policies of the course, and have appropriately and completely documented any and all use of AI-tools.** I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies.
- Work missing the honesty pledge **may not be graded!**



Honesty Pledges, II

- Larger reports must contain the following text:
  - I, `[your name]`, pledge that this report is completely my own work, and that I did not take, borrow or steal any portions from any other person — real or artificial — and that I did not allow any other person to use, have, borrow or steal portions of my report. Any and all references I used are clearly cited in the text. **I have adhered to the AI-policies of the course, and have appropriately and completely documented any and all use of AI-tools.** I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies. `[Your signature]`.
- Work missing the honesty pledge `may not be graded!`

Math 543: Computer Resources

You need access to a computing environment in which to write your code; — you may want to use any combination of Matlab (for quick prototyping and short homework assignments) and other languages; *e.g.* Python, C/C++ or Fortran (or something completely different, like Java or M$-D$^\flat$).

Free C/C++ and Fortran compilers are available for Linux/UNIX.

SDSU students can download a copy of matlab from

`https://www.mathworks.com/academia/tah-portal/san-diego-state-university-1108597.html`

[Licensing Subject to Change With Minimal Notice]

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Math 543: Introduction — What you should know already

Prerequisite: `Math` 340

340 $\Rightarrow$ **Programming in Mathematics**

- Introduction to programming in mathematics. Modelling, problem solving, visualization..

Prerequisite: `Math` 254 **or** `Math` 342A **or** `AE` 280

254 $\cap$ 342A $\cap$ `AE` 280 $\Rightarrow$ **Basic Linear Algebra**

- Vectors, Matrices, Eigenvalues and Eigenvectors

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \qquad A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \dots & \vec{a}_n \end{bmatrix}$$

Math 543: Introduction — Why???

Solution of linear systems and eigenvalue problems show up in many applications in applied & computational mathematics / sciences / engineering.

Although we probably know about Gaussian Elimination for solving

$$\vec{x} = A^{-1}\vec{b}, \quad \text{where } A \in \mathbb{R}^{n\times n},\ \vec{x}, \vec{b} \in \mathbb{R}^n$$

in infinite precision (by hand), finding this solution (or at least a good approximation thereof) in **finite precision** (*i.e.* on a computer) is sometimes a challenge — especially if we need the solution fast...

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Math 543: Introduction — Why???

The computational complexity (number of operations needed) for Gaussian Elimination is $\mathcal{O}(n^3)$, which is quite slow as n grows "large."

Applications (sources of Numerical Linear Algebra problems):

- Solution of partial differential equations (PDEs)
- Optimization (Operations Research)
- Model Analysis and Fitting (Least Squares)
- Image Processing
- Protein Folding
- DNA sequencing, etc. etc. etc.
- **Data Science, Machine Learning, AI, etc...**



Math 543: Introduction — What We Will Discuss

$$\mathbf{A\tilde{x} = \tilde{b}}, \quad \mathbf{A\tilde{x} = \lambda\tilde{x}}, \quad \mathbf{Q^T AQ = \Lambda} = \operatorname{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n), \quad \mathbf{A = U\Sigma V}^*$$

- QR-Factorization / Least Squares
- The Singular Value Decomposition
- Conditioning and Stability
- Gaussian Elimination, Pivoting

  $\Rightarrow$ LU- and Cholesky-factorization
- Eigenvalue Problems
- Iterative Methods

  $\Rightarrow$ Arnoldi, GMRES, Lanczos, Conjugate Gradients

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