Numerical Matrix Analysis
Notes #1 — Introduction

Peter Blomgren
⟨blomgren@sdsu.edu⟩
Department of Mathematics and Statistics
Dynamical Systems Group
Computational Sciences Research Center
San Diego State University
San Diego, CA 92182-7720
http://terminus.sdsu.edu/

Spring 2023
(Revised: February 22, 2023)

1. Introduction — (1/26)

The Professor
The Class — Overview
The Class...
Introduction
Academic Life
Non-Academic Life
Contact Information, Office Hours

Academic Life
MSc

- MSc. Engineering Physics, Royal Institute of Technology (KTH), Stockholm, Sweden. Thesis Advisers: Michael Benedicks, Department of Mathematics KTH, and Erik Aurell, Stockholm University, Department of Mathematics. Thesis Topic: “A Renormalization Technique for Families with Flat Maxima.”

Figure: Bifurcation diagram for the family $f_{a,1}$ [BLOMGREN-1994]

PhD


Figure: The noisy (SNR = 4.62 dB), and recovered space curves. Notice how the edges are recovered. [BLOMGREN-1998]
Fun Times... $\leftrightarrow$ Endurance Sports

Pre-Pandemic...

Triathlons:
- (13) Ironman distance (2.4 + 112 + 26.2)  [PR] 11:48:57
  5:14:20
- (16) Half Ironman distance

Running
- (1) 100k Race (62.1 miles)  15:37:46 (15:05/mi)
- (1) Trail Double-marathon (52 miles)  10:59:00 (12:32/mi)
- (5) Trail 50-mile races  9:08:46 (10:59/mi)
- (8) Trail 50k (31 mile) races  5:20:57 (10:20/mi)
- (16) Road/Trail Marathons  3:26:19 (7:52/mi)
- (30) Road/Trail Half Marathons  1:35:00 (7:15/mi)
The Post(?) Pandemic Reality

- Spring 2020: We went online part-way thru the semester
- Spring 2021: Zoom-U
- Spring 2022: Masked-U — not a good semester (many reasons)
- Spring 2023: The New Normal™... some modifications.

During COVID we all de-socialized to some extent; and there is definitely a “learning gap.”

There is sometimes a tendency to see all this silly learning and class work as unnecessary obstacles to getting a degree.

... of least resistance and have Uncle Google, Aunt Wiki, Scuzzy Cousin Chegg, Skynet, or ChatGPT do all the heavy lifting...

We need to remind ourselves that education is about developing skills and processes, definitely not just about “having The Answer.” Understanding

- how to get The Answer;
- how to validate The Answer;
- what The Answer means; and
- how The Answer potentially is useful;

are often the bigger and more important lessons.

It is “unlikely” that you, in real life, will be asked to differentiate $\cos(\sin(\tan(\ln(x))))$, or write the 55,000,001st analysis of Fyodor Dostoevsky’s “Crime and Punishment,” or single-handedly implement the Singular Value Decomposition... but acquiring the skills to perform these tasks are arguable useful.

Math 543: Literature

“Required” —


“Required” — (Supplemental)

Class notes and class web-page.

* SIAM members receive special pricing (30% off). SIAM student membership is free.
Math 543: Literature

Everything You Ever Wanted to Know, but Were Afraid to Ask...

“Optional” — (Classic, Comprehensive Reference)


“Optional” — (Comprehensive Reference)


* SIAM members receive special pricing (30% off). SIAM student membership is free.

Math 543: Introduction — Grading etc.

50% Homework: both theoretical, and implementation (programming) — Recommended languages: Python, Matlab, C/C++, or Fortran; however anything goes: 6510 assembler, Java, M$-D$, Haskell...

25% Midterm: [½ Take-Home, and ½ In-Class].

25% Final: [½ Take-Home, and ½ In-Class].

Expectations and Procedures, I

- Most class attendance is “OPTIONAL” — Homework and announcements will be posted on the class web page. If/when you attend class:
  - Please be on time.
  - Please pay attention.
  - Please turn off mobile phones.
  - Please be courteous to other students and the instructor.
  - Abide by university statutes, and all applicable local, state, and federal laws.
Expectations and Procedures, II

- Please, turn in assignments on time. (The instructor reserves the right not to accept late assignments.)
- The instructor will make special arrangements for students with documented learning disabilities and will try to make accommodations for other unforeseen circumstances, e.g. illness, personal/family crises, etc. in a way that is fair to all students enrolled in the class. Please contact the instructor EARLY regarding special circumstances.
- Students are expected and encouraged to ask questions in class!
- Students are expected and encouraged to to make use of office hours! If you cannot make it to the scheduled office hours: contact the instructor to schedule an appointment!

Peter Bloomgren (blomgren@sdsu.edu) 1. Introduction — (17/26)

Expectations and Procedures, III

- Missed midterm exams: Don’t miss exams! The instructor reserves the right to schedule make-up exams, make such exams oral presentation, and/or base the grade solely on other work (including the final exam).
- Missed final exam: Don’t miss the final! Contact the instructor ASAP or a grade of incomplete or F will be assigned.

- Academic honesty: submit your own work — but feel free to discuss homework with other students in the class!

Peter Bloomgren (blomgren@sdsu.edu) 1. Introduction — (18/26)

Late HW Policy

- Assignments accepted up to 24 hours after original deadline, with a 10% penalty.
- Further extensions will only be granted in extreme, well-documented, circumstances.

Peter Bloomgren (blomgren@sdsu.edu) 1. Introduction — (19/26)

Honesty Pledges, I

- The following Honesty Pledge must be included in all programs you submit (as part of homework and/or projects):
  - I, [your name], pledge that this program is completely my own work, and that I did not take, borrow or steal code from any other person — real or artificial — and that I did not allow any other person to use, have, borrow or steal portions of my code. I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies.

- Work missing the honesty pledge may not be graded!

Peter Bloomgren (blomgren@sdsu.edu) 1. Introduction — (20/26)
Honesty Pledges, II

- Larger reports must contain the following text:
  - I, [your name], pledge that this report is completely my own work, and that I did not take, borrow or steal any portions from any other person — real or artificial — and that I did not allow any other person to use, have, borrow or steal portions of my report. Any and all references I used are clearly cited in the text. I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies. [Your signature].
  - Work missing the honesty pledge may not be graded!

Math 543: Introduction — What you should know already

**Prerequisite:** Math 340

340 ⇒ Programming in Mathematics
- Introduction to programming in mathematics. Modelling, problem solving, visualization.

**Prerequisite:** Math 254 or Math 342A or AE 280

254 ∩ 342A ∩ AE 280 ⇒ Basic Linear Algebra
- Vectors, Matrices, Eigenvalues and Eigenvectors

\[ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = \begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \\ \vdots \\ \bar{a}_n \end{bmatrix} \]

Math 543: Introduction — Why???

Solution of linear systems and eigenvalue problems show up in many applications in applied & computational mathematics / sciences / engineering.

Although we probably know about Gaussian Elimination for solving

\[ \mathbf{x} = A^{-1}\mathbf{b}, \quad \text{where } A \in \mathbb{R}^{n \times n}, \quad \mathbf{x}, \mathbf{b} \in \mathbb{R}^n \]

in infinite precision (by hand), finding this solution (or at least a good approximation thereof) in finite precision (i.e. on a computer) is sometimes a challenge — especially if we need the solution fast...
Math 543: Introduction — Why???

The computational complexity (number of operations needed) for Gaussian Elimination is $O(n^3)$, which is quite slow as $n$ grows "large."

Applications (sources of Numerical Linear Algebra problems):
- Solution of partial differential equations (PDEs)
- Optimization (Operations Research)
- Model Analysis and Fitting (Least Squares)
- Image Processing
- Protein Folding
- DNA sequencing, etc. etc. etc.
- Data Science, Machine Learning, AI, etc...

Math 543: Introduction — What We Will Discuss

$$A\tilde{x} = \tilde{b}, \quad A\tilde{x} = \lambda \tilde{x}, \quad Q^T AQ = \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n), \quad A = U\Sigma V^*$$

- QR-Factorization / Least Squares
- The Singular Value Decomposition
- Conditioning and Stability
- Gaussian Elimination, Pivoting
  $$\Rightarrow \text{LU- and Cholesky-factorization}$$
- Eigenvalue Problems
- Iterative Methods
  $$\Rightarrow \text{Arnoldi, GMRES, Lanczos, Conjugate Gradients}$$