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# Mathematics Personae

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Johann Carl Friedrich Gauss (30 April 1777 — 23 Feb 1855)

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At the age of seven, Carl Friedrich Gauss started elementary school, and his potential was noticed almost immediately. His teacher, Büttner, and his assistant, Martin Bartels, were amazed when Gauss summed the integers from 1 to 100 instantly by spotting that the sum was 50 pairs of numbers each pair summing to 101.

In 1788 Gauss began his education at the Gymnasium with the help of Büttner and Bartels, where he learned High German and Latin. After receiving a stipend from the Duke of Brunswick-Wolfenbüttel, Gauss entered Brunswick Collegium Carolinum in 1792. At the academy Gauss independently discovered Bode's law, the binomial theorem and the arithmetic-geometric mean, as well as the law of quadratic reciprocity and the prime number theorem.

In 1795 Gauss left Brunswick to study at Göttingen University. Gauss's teacher there was Kästner, whom Gauss often ridiculed. His only

known friend amongst the students was Farkas Bolyai. They met in 1799 and corresponded with each other for many years.

Gauss left Göttingen in 1798 without a diploma, but by this time he had made one of his most important discoveries — the construction of a regular 17-gon by ruler and compasses. This was the most major advance in this field since the time of Greek mathematics and was published as Section VII of Gauss's famous work, *Disquisitiones Arithmeticae*.

Gauss returned to Brunswick where he received a degree in 1799. After the Duke of Brunswick had agreed to continue Gauss's stipend, he requested that Gauss submit a doctoral dissertation to the University of Helmstedt. He already knew Pfaff, who was chosen to be his adviser. Gauss's dissertation was a discussion of the fundamental theorem of algebra.

With his stipend to support him, Gauss did not need to find a job so devoted himself to research. He published the book *Disquisitiones Arithmeticae* in the summer of 1801. There were seven sections, all but the last section, referred to above, being devoted to number theory.

In June 1801, Zach, an astronomer whom Gauss had come to know two or three years previously, published the orbital positions of Ceres, a new "small planet" which was discovered by G. Piazzi, an Italian astronomer on 1 January, 1801. Unfortunately, Piazzi had only been able to observe 9 degrees of its orbit before it disappeared behind the Sun. Zach published several predictions of its position, including one by Gauss which differed greatly from the others. When Ceres was rediscovered by Zach on 7 December 1801 it was almost exactly where Gauss had predicted. Although

he did not disclose his methods at the time, Gauss had used his least squares approximation method.

In June 1802 Gauss visited Olbers who had discovered Pallas in March of that year and Gauss investigated its orbit. Olbers requested that Gauss be made director of the proposed new observatory in Göttingen, but no action was taken. Gauss began corresponding with Bessel, whom he did not meet until 1825, and with Sophie Germain.

Gauss married Johanna Ostoff on 9 October, 1805. Despite having a happy personal life for the first time, his benefactor, the Duke of Brunswick, was killed fighting for the Prussian army. In 1807 Gauss left Brunswick to take up the position of director of the Göttingen observatory.

Gauss arrived in Göttingen in late 1807. In 1808 his father died, and a year later Gauss's wife Johanna died after giving birth to their second son, who was to die soon after her. Gauss was shattered and wrote to Olbers asking him to give him a home for a few weeks,

to gather new strength in the arms of  
your friendship — strength for a life  
which is only valuable because it be-  
longs to my three small children.

Gauss was married for a second time the next year, to Minna the best friend of Johanna, and although they had three children, this marriage seemed to be one of convenience for Gauss.

Gauss's work never seemed to suffer from his personal tragedy. He published his second book, *Theoria motus corporum coelestium in sectionibus conicis Solem ambientium*, in 1809, a major two volume treatise on the motion of celestial bodies. In the first volume he discussed differential equations, conic sections and elliptic orbits, while in the second volume, the main part of the work, he showed how to estimate and then to refine the estimation of a planet's orbit. Gauss's contributions to theoretical astronomy stopped after 1817, although he went on making observations until the age of 70.

Much of Gauss's time was spent on a new observatory, completed in 1816, but he still found

the time to work on other subjects. His publications during this time include *Disquisitiones generales circa seriem infinitam*, a rigorous treatment of series and an introduction of the hypergeometric function, *Methodus nova integralium valores per approximationem inveniendi*, a practical essay on approximate integration, *Bestimmung der Genauigkeit der Beobachtungen*, a discussion of statistical estimators, and *Theoria attractionis corporum sphaeroidicorum ellipticorum homogeneorum methodus nova tractata*. The latter work was inspired by geodesic problems and was principally concerned with potential theory. In fact, Gauss found himself more and more interested in geodesy in the 1820s.

Gauss had been asked in 1818 to carry out a geodesic survey of the state of Hanover to link up with the existing Danish grid. Gauss was pleased to accept and took personal charge of the survey, making measurements during the day and reducing them at night, using his extraordinary mental capacity for calculations. He regularly wrote to Schumacher, Olbers and Bessel, reporting on his progress and discussing problems.

Because of the survey, Gauss invented the heliotrope which worked by reflecting the Sun's rays using a design of mirrors and a small telescope. However, inaccurate base lines were used for the survey and an unsatisfactory network of triangles. Gauss often wondered if he would have been better advised to have pursued some other occupation but he published over 70 papers between 1820 and 1830.

In 1822 Gauss won the Copenhagen University Prize with *Theoria attractionis...* together with the idea of mapping one surface onto another so that the two *are similar in their smallest parts*. This paper was published in 1825 and led to the much later publication of *Untersuchungen über Gegenstände der Höheren Geodäsie* (1843 and 1846). The paper *Theoria combinationis observationum erroribus minimis obnoxiae* (1823), with its supplement (1828), was devoted to mathematical statistics, in particular to the least squares method.

From the early 1800s Gauss had an interest in the question of the possible existence of a non-Euclidean geometry. He discussed this topic at length with Farkas Bolyai and in his correspondence with Gerling and Schumacher. In a book review in 1816 he discussed proofs which deduced the axiom of parallels from the other Euclidean axioms, suggesting that he believed in the existence of non-Euclidean geometry, although he was rather vague. Gauss confided in Schumacher, telling him that he believed his reputation would suffer if he admitted in public that he believed in the existence of such a geometry.

In 1831 Farkas Bolyai sent to Gauss his son János Bolyai's work on the subject. Gauss replied

to praise it would mean to praise myself.

Again, a decade later, when he was informed of Lobachevsky's work on the subject, he praised its "genuinely geometric" character, while in a letter to Schumacher in 1846, states that he

had the same convictions for 54 years

indicating that he had known of the existence of a non-Euclidean geometry since he was 15 years of age (this seems unlikely).

Gauss had a major interest in differential geometry, and published many papers on the subject. *Disquisitiones generales circa superficies curva* (1828) was his most renowned work in this field. In fact, this paper rose from his geodesic interests, but it contained such geometrical ideas as Gaussian curvature. The paper also includes Gauss's famous *theorema egregium*:

If an area in  $E^3$  can be developed (i.e. mapped isometrically) into another area of  $E^3$ , the values of the Gaussian curvatures are identical in corresponding points.

The period 1817–1832 was a particularly distressing time for Gauss. He took in his sick mother in 1817, who stayed until her death in 1839, while

he was arguing with his wife and her family about whether they should go to Berlin. He had been offered a position at Berlin University and Minna and her family were keen to move there. Gauss, however, never liked change and decided to stay in Göttingen. In 1831 Gauss's second wife died after a long illness.

In 1831, Wilhelm Weber arrived in Göttingen as physics professor filling Tobias Mayer's chair. Gauss had known Weber since 1828 and supported his appointment. Gauss had worked on physics before 1831, publishing *über ein neues allgemeines Grundgesetz der Mechanik*, which contained the principle of least constraint, and *Principia generalia theoriae figurae fluidorum in statu aequilibrum* which discussed forces of attraction. These papers were based on Gauss's potential theory, which proved of great importance in his work on physics. He later came to believe his potential theory and his method of least squares provided vital links between science and nature.

In 1832, Gauss and Weber began investigating the theory of terrestrial magnetism after Alexander von Humboldt attempted to obtain Gauss's assistance in making a grid of magnetic observation points around the Earth. Gauss was excited by this prospect and by 1840 he had written three important papers on the subject: *Intensitas vis magneticae terrestris ad mensuram absolutam revocata* (1832), *Allgemeine Theorie des Erdmagnetismus* (1839) and *Allgemeine Lehrsätze in Beziehung auf die im verkehrten Verhältnisse des Quadrats der Entfernung wirkenden Anziehungs- und Abstossungskräfte* (1840). These papers all dealt with the current theories on terrestrial magnetism, including Poisson's ideas, absolute measure for magnetic force and an empirical definition of terrestrial magnetism. Dirichlet's principle was mentioned without proof.

*Allgemeine Theorie...* showed that there can only be two poles in the globe and went on to prove an important theorem, which concerned the determination of the intensity of the horizontal component of the magnetic force along with the angle of inclination. Gauss used the Laplace equation to

aid him with his calculations, and ended up specifying a location for the magnetic South pole.

Humboldt had devised a calendar for observations of magnetic declination. However, once Gauss's new magnetic observatory (completed in 1833 — free of all magnetic metals) had been built, he proceeded to alter many of Humboldt's procedures, not pleasing Humboldt greatly. However, Gauss's changes obtained more accurate results with less effort.

Gauss and Weber achieved much in their six years together. They discovered Kirchhoff's laws, as well as building a primitive telegraph device which could send messages over a distance of 5000 ft. However, this was just an enjoyable pastime for Gauss. He was more interested in the task of establishing a world-wide net of magnetic observation points. This occupation produced many concrete results. The *Magnetischer Verein* and its journal were founded, and the atlas of geomagnetism was published, while Gauss and Weber's own journal in which their results were published ran from 1836 to 1841.

In 1837, Weber was forced to leave Göttingen when he became involved in a political dispute and, from this time, Gauss's activity gradually decreased. He still produced letters in response to fellow scientists' discoveries usually remarking that he had known the methods for years but had never felt the need to publish. Sometimes he seemed extremely pleased with advances made by other mathematicians, particularly that of Eisenstein and of Lobachevsky.

Gauss spent the years from 1845 to 1851 updating the Göttingen University widow's fund. This work gave him practical experience in financial matters, and he went on to make his fortune through shrewd investments in bonds issued by private companies.

Two of Gauss's last doctoral students were Moritz Cantor and Dedekind. Dedekind wrote a fine description of his supervisor

... usually he sat in a comfortable atti-

tude, looking down, slightly stooped, with hands folded above his lap. He spoke quite freely, very clearly, simply and plainly: but when he wanted to emphasize a new viewpoint ... then he lifted his head, turned to one of those sitting next to him, and gazed at him with his beautiful, penetrating blue eyes during the emphatic speech. ... If he proceeded from an explanation of principles to the development of mathematical formulas, then he got up, and in a stately very upright posture he wrote on a blackboard beside him in his peculiarly beautiful handwriting: he always succeeded through economy and deliberate arrangement in making do with a rather small space. For numerical examples, on whose careful completion he placed special value, he brought along the requisite data on little slips of paper.

Gauss presented his golden jubilee lecture in 1849, fifty years after his diploma had been granted by Helmstedt University. It was appropriately a variation on his dissertation of 1799. From the mathematical community only Jacobi and Dirichlet were present, but Gauss received many messages and honors.

From 1850 onwards Gauss's work was again nearly all of a practical nature although he did approve Riemann's doctoral thesis and heard his probationary lecture. His last known scientific exchange was with Gerling. He discussed a modified Foucault pendulum in 1854. He was also able to attend the opening of the new railway link between Hanover and Göttingen, but this proved to be his last outing. His health deteriorated slowly, and Gauss died in his sleep early in the morning of 23 February, 1855.

Article by: J J O'Connor and E F Robertson.