Mathematics Personae

Issai Schur (10 Jan 1875 — 10 Jan 1941)

Although Issai Schur was born in Mogilyov on the Dnieper, he spoke German without a trace of an accent, and nobody even guessed that it was not his first language. He went to Latvia at the age of 13 and there he attended the Gymnasium in Libau, now called Liepaja.

In 1894 Schur entered the University of Berlin to read mathematics and physics. Frobenius was one of his teachers and he was to greatly influence Schur and later to direct his doctoral studies. Frobenius and Burnside had been the two main founders of the theory of representations of groups as groups of matrices. This theory proved a very powerful tool in the study of groups and Schur was to learn the foundations of this subject from Frobenius. Schur then made major steps forward, both in work of his own and work done in collaboration with Frobenius.

In 1901 Schur obtained his doctorate with a thesis which examined rational representations of the general linear group over the complex field. Functions which Schur introduced in his thesis are today called S-functions, where the S stands for Schur. Interest in the results of Schur's thesis continues today; for example J A Green published an account of these results in a modern setting in 1980.

In 1903 Schur became a lecturer at Berlin University and then, from 1911 until 1916, he held a professorship in mathematics at the University of Bonn. He returned to Berlin in 1916 and there he built his famous school and spent most of the rest of his life there. He was promoted to full professor in Berlin in 1919, three years after he returned there, and he held this chair until he was dismissed by the Nazis in 1935.

Schur is mainly known for his fundamental work on the representation theory of groups but he also worked in number theory, analysis and other topics described below. Between 1904 and 1907 he worked on projective representations of groups and group characters. One of the most fundamental results which he discovered at this time is today called Schur's Lemma.

In a series of papers he introduced the concept now known as the 'Schur multiplier'. This is an extremely important abstract concept which arose from the concrete problems that Schur was studying. Much later, in 1949, Eilenberg and Mac Lane defined cohomology groups. They were unaware at that time that the second cohomology group with coefficients in the nonzero complex numbers is the Schur multiplier, and therefore that Schur had made some of the first steps forty years earlier.

Around 1914 Schur's interest in representations of groups was put to one side while he worked on other topics but, around 1925, developments

in theoretical physics showed that group representations were of fundamental importance in that subject. Schur returned to work on representation theory with renewed vigor and he was able to complete the program of research begun in his doctoral dissertation and give a complete description of the rational representations of the general linear group.

Schur was also interested in reducibility, location of roots and the construction of the Galois group of classes of polynomials such as Laguerre and Hermite polynomials. In [1] an indication of the other topics that Schur worked on is given:-

> First there was pure group theory, in which Schur adopted the surprising approach of proving without the aid of characters, theorems that had previously been demonstrated only by that means.
> Second, he worked in the field of matrices.
> Third, he handled algebraic equations, sometimes proceeding to the evaluation of roots, and sometimes treating the so-called equation without affect, that is, with symmetric Galois groups. He was also the first to give examples of equations with alternating Galois groups.
> Fourth, he worked in number theory;
> Fifth, in divergent series;
> Sixth, in integral equations;
> and lastly in function theory.

The school which Schur built at Berlin was of major importance not only for the representation theory of groups but, as indicated above, for other areas of mathematics. The school partly worked through the Schur's lecturing [7]:-

> ...there are [many] mathematicians who went to Schur's lectures and seminars in Berlin and were strongly influenced by him...

The school also worked with collaborations [1]:-

> A lively interchange with many colleagues led Schur to contribute important memoirs .... Some of these were published as collaborations with other authors, although publications with dual authorship were almost unheard of at that time.

This school was certainly the most coherent and influential group of mathematicians in Berlin, and among the most important in all of Germany. Schur's charismatic leadership inspired those around him to push forward with research on group representations. Schur's own impressive contributions were extended by his students in a number of different directions. They worked on topics such as soluble groups, combinatorics, and matrix theory.

Among the students who completed their doctorates under Schur were Richard Brauer, Alfred Brauer (Richard Brauer's brother), Robert Frucht, Bernhard Neumann, Richard Rado, and Helmut Wielandt. There were others who worked under Schur such as Kurt Hirsch, Walter Ledermann, Hanna Neumann and Menahem Max Schiffer.

Ledermann in [7] describes Schur as a teacher:-

> Schur was a superb lecturer. His lectures were meticulously prepared... [and] were exceedingly popular. I remember attending his algebra course which was held in a lecture theater filled with about 400 students. Sometimes, when I had to be content with a seat at the back of the lecture theater, I used a pair of opera glasses to get at least a glimpse of the speaker.

In 1922 Schur was elected to the Prussian Academy, proposed by Planck, the secretary of the Academy. Planck's address which listed Schur's outstanding achievements had been written by Frobenius, at least five years earlier, as Frobenius died in 1917.

From 1933 events in Germany made Schur's life increasingly difficult. Hirsch spoke of the events of 1 April 1933 when posters carried the message 'Germans defend yourselves against Jewish atrocity propaganda : buy only at German shops':-

> That was the so-called 'Boycott Day', the day on which Jewish shops were boycotted and Jewish professors and lecturers were not allowed to enter the university. Everybody who was there had to make a little speech about the rejuvenation of Germany etc. And Bieberbach did this quite nicely and then he said 'A drop of remorse falls into my joy because my dear friend and colleague Schur is not allowed to be among us today'.

On 7 April 1933 the Nazis passed a law which, under clause three, ordered the retirement of civil servants who were not of Aryan descent, with exemptions for participants in World War I and pre-war officials. Schur had held an appointment before World War I which should have qualified him as a civil servant, but the facts were not allowed to get in the way, and he was 'retired'. Schiffer wrote [8]:-

> When Schur's lectures were canceled there was an outcry among the students and professors, for Schur was respected and very well liked. The next day Erhard Schmidt started his lecture with a protest against this dismissal and even Bieberbach, who later made himself a shameful reputation as a Nazi, came out in Schur's defense. Schur went on quietly with his work on algebra at home.

Schur saw himself as a German, not a Jew, and could not comprehend the persecution and humiliation he suffered under the Nazis. In fact Schur's dismissal was revoked and he was able to carry out some of his duties for a while. By November 1933 when Walter Ledermann took his *Staatsexamen* he was examined by Schur together with Bieberbach who was wearing Nazi uniform.

There were invitations to Schur to go to the United States and to Britain but he declined them all, unable to understand how a German was not welcome in Germany. For example Ledermann obtained a scholarship to go to St Andrews in Scotland in the spring of 1934 and he tried unsuccessfully to persuade Schur to join him in St Andrews.

Schur continued to suffer the humiliation that was heaped on him. Schiffer recalls an event in [8] relating to Schur's 60th birthday on 10 January 1935:-

> Schur told me that the only person at the Mathematical Institute in Berlin who was kind to him was Grunsky, then a young lecturer. Long after the war, I talked to Grunsky about that remark and he literally started to cry: "You know what I did? I sent him a postcard to congratulate him on his sixtieth birthday. I admired him so much and was very respectful in that card. How lonely he must have been to remember such a small thing."

Later in 1935 Schur was dismissed from his chair in Berlin but he continued to work there suffering great hardship and difficulties. Alfred Brauer writes in [6]:-

> When Landau died in February 1938, Schur was supposed to give an address at his funeral. For that reason he was in need of some mathematical details from the literature. He asked me to help him in this matter. Of course I was not allowed to use the library of the mathematical institute which I had built up over many years. Finally I got an exemption for a week and could use the library of the Prussian Staatsbibliothek for a fee. ... So I could answer at least some of Schur's questions.

Pressure was put on Schur to resign from the Prussian Academy to which he had been honored to be elected in 1922. On 29 March 1938 Bieberbach wrote below Schur's signature on a document of the Prussian Academy:-

> I find it surprising that Jews are still members of academic commissions.

Just over a week later, on 7 April 1938, Schur resigned from Commissions of the Academy. However, the pressure on him continued and later that year he resigned completely from the Academy.

Schur left Germany for Palestine in 1939, broken in mind and body, having the final humiliation of being forced to find a sponsor to pay the 'Reichs flight tax' to allow him to leave Germany. Without sufficient funds to live in Palestine he was forced to sell his beloved academic books to the Institute for Advanced Study in Princeton. He died two years later on his 66th birthday.

Article by: J J O'Connor and E F Robertson.

References:

[1] Biography in *Dictionary of Scientific Biography* (New York 1970-1990).

[6] A Brauer, Gedenkrede auf Issai Schur, in A Brauer and H Rohrbach (eds.), *I Schur, Gesammelte Abhandlungen* (Berlin, 1973), v-xiii.

[7] W Ledermann, Issai Schur and his school in Berlin, *Bull. London Math. Soc.* **15** (1983), 97-106.

[8] M M Schiffer, Issai Schur : Some personal reminiscences, in H Begehr (ed.), *Mathematik in Berlin : Geschichte und Dokumentation* (Aachen, 1998).