Mathematics Personae

Brook Taylor (18 Aug 1685 — 29 Dec 1731)

Brook Taylor's father was John Taylor and his mother was Olivia Tempest. John Taylor was the son of Natheniel Taylor who was recorder of Colchester and a member representing Bedfordshire in Oliver Cromwell's Assembly, while Olivia Tempest was the daughter of Sir John Tempest. Brook was, therefore, born into a family which was on the fringes of the nobility and certainly they were fairly wealthy.

Taylor was brought up in a household where his father ruled as a strict disciplinarian, yet he was a man of culture with interests in painting and music. Although John Taylor had some negative influences on his son, he also had some positive ones, particularly giving his son a love of music and painting. Brook Taylor grew up not only to be an accomplished musician and painter, but he applied his mathematical skills to both these areas later in his life.

As Taylor's family were well off they could afford to have private tutors for their son and in fact this home education was all that Brook enjoyed before entering St John's College Cambridge on 3 April 1703. By this time he had a good grounding in classics and mathematics. At Cambridge Taylor became highly involved with mathematics. He graduated with an LL.B. in 1709 but by this time he had already written his first important mathematics paper (in 1708) although it would not be published until 1714. We know something of the details of Taylor thoughts on various mathematical problems from letters he exchanged with Machin and Keill beginning in his undergraduate years.

In 1712 Taylor was elected to the Royal Society. This was on the 3 April, and clearly it was an election based more on the expertise which Machin, Keill and others knew that Taylor had, rather than on his published results. For example Taylor wrote to Machin in 1712 providing a solution to a problem concerning Kepler's second law of planetary motion. Also in 1712 Taylor was appointed to the committee set up to adjudicate on whether the claim of Newton or of Leibniz to have invented the calculus was correct.

The paper we referred to above as being written in 1708 was published in the *Philosophical Transactions of the Royal Society* in 1714. The paper gives a solution to the problem of the center of oscillation of a body, and it resulted in a priority dispute with Johann Bernoulli. We shall say a little more below about disputes between Taylor and Johann Bernoulli. Returning to the paper, it is a mechanics paper which rests heavily on Newton's approach to the differential calculus.

The year 1714 also marks the year in which Taylor was elected Secretary to the Royal Society. It was a position which Taylor held from 14 January of that year until 21 October 1718 when he resigned, partly for health reasons, partly due to his lack of interest in the rather demanding position. The period during which Taylor was Secretary to the Royal Society does mark what must be considered his most mathematically productive time. Two books which appeared in 1715, *Methodus incrementorum directa et inversa* and *Linear Perspective* are extremely important in the history of mathematics. Second editions would appear in 1717 and 1719 respectively. We discuss the content of these works in some detail below.

Taylor made several visits to France. These were made partly for health reasons and partly to visit the friends he had made there. He met Pierre Rémond de Montmort and corresponded with him on various mathematical topics after his return. In particular they discussed infinite series and probability. Taylor also corresponded with de Moivre on probability and at times there was a three-way discussion going on between these mathematicians.

Between 1712 and 1724 Taylor published thirteen articles on topics as diverse as describing experiments in capillary action, magnetism and thermometers. He gave an account of an experiment to discover the law of magnetic attraction (1715) and an improved method for approximating the roots of an equation by giving a new method for computing logarithms (1717). His life, however, suffered a series of personal tragedies beginning around 1721. In that year he married Miss Brydges from Wallington in Surrey. Although she was from a good family, it was not a family with money and Taylor's father strongly objected to the marriage. The result was that relations between Taylor and his father broke down and there was no contact between father and son until 1723. It was in that year that Taylor's wife died in childbirth. The child, which would have been their first, also died.

After the tragedy of losing his wife and child, Taylor returned to live with his father and relations between the two were repaired. Two years later, in 1725, Taylor married again to Sabetta Sawbridge from Olantigh in Kent. This marriage had the approval of Taylor's father who died four years later on 4 April 1729. Taylor inherited his father's estate of Bifons but further tragedy was to strike when his second wife Sabetta died in childbirth in the following year. On this occasion the child, a daughter Elizabeth, did survive.

Taylor added to mathematics a new branch now called the "calculus of finite differences", invented integration by parts, and discovered the celebrated series known as Taylor's expansion. These ideas appear in his book *Methodus incrementorum directa et inversa* of 1715 referred to above. In fact the first mention by Taylor of a version of what is today called Taylor's Theorem appears in a letter which he wrote to Machin on 26 July 1712. In this letter Taylor explains carefully where he got the idea from.

It was, wrote Taylor, due to a comment that Machin made in Child's Coffeehouse when he had commented on using "Sir Isaac Newton's series" to solve Kepler's problem, and also using "Dr Halley's method of extracting roots" of polynomial equations. There are, in fact, two versions of Taylor's Theorem given in the 1715 paper which to a modern reader look equivalent but which, the author of [3] argues convincingly, were differently motivated. Taylor initially derived the version which occurs as Proposition 11 as a generalization of Halley's method of approximating roots of the Kepler equation, but soon discovered that it was a consequence of the Bernoulli series. This is the version which was inspired by the Coffeehouse conversation described above. The second version occurs as Corollary 2 to Proposition 7 and was thought of as a method of expanding solutions of fluxional equations in infinite series.

We must not give the impression that this result was one which Taylor was the first to discover. James Gregory, Newton, Leibniz, Johann Bernoulli and de Moivre had all discovered variants of Taylor's Theorem. Gregory, for example, knew that

$$\arctan x = x - x^3/3 + x^5/5 - x^7/7 + ...$$

and his methods are discussed in [4]. The differences in Newton's ideas of Taylor series and those of Gregory are discussed in [6]. All of these mathematicians had made their discoveries independently, and Taylor's work was also independent of that of the others. The importance of Taylor's Theorem remained unrecognized until 1772 when Lagrange proclaimed it the basic principle of the differential calculus. The term "Taylor's series" seems to have used for the first time by Lhuilier in 1786.

There are other important ideas which are contained in the *Methodus incrementorum directa et inversa* of 1715 which were not recognized as important at the time. These include singular solutions to differential equations, a change of variables formula, and a way of relating the derivative of a function to the derivative of the inverse function. Also contained is a discussion on vibrating strings, an interest which almost certainly come from Taylor's early love of music.

Taylor, in his studies of vibrating strings was not attempting to establish equations of motion, but was considering the oscillation of a flexible string in terms of the isochrony of the pendulum. He tried to find the shape of the vibrating string and the length of the isochronous pendulum rather than to find its equations of motion. Further discussion of these ideas is given in [5].

Taylor also devised the basic principles of perspective in *Linear Perspective* (1715). The second edition has a different title, being called *New principles of linear perspective.* The work gives first general treatment of vanishing points. Taylor had a highly mathematical approach to the subject and made no concessions to artists who should have found the ideas of fundamental importance to them. At times it is very difficult for even a mathematician to understand Taylor's results. The phrase "linear perspective" was invented by Taylor in this work and he defined the vanishing point of a line, not parallel to the plane of the picture, as the point where a line through the eye parallel to the given line intersects the plane of the picture. He also defined the vanishing line to a given plane, not parallel to the plane of the picture, as the intersection of the plane through the eye parallel to the given plane. He did not invent the terms vanishing point and vanishing line, but he was one of the first to stress their importance. The main theorem in Taylor's theory of linear perspective is that the projection of a straight line not parallel to the plane of the picture passes through its intersection and its vanishing point.

There is also the interesting inverse problem which is to find the position of the eye in order to see the picture from the viewpoint that the artist intended. Taylor was not the first to discuss this inverse problem but he did make innovative contributions to the theory of such perspective problems. One could certainly consider this work as laying the foundations for the theory of descriptive and projective geometry.

Taylor challenged the "non-English mathematicians" to integrate a certain differential. One has to see this challenge as part of the argument between the Newtonians and the Leibnitzians. Conte in [2] discusses the answers given by Johann Bernoulli and Giulio Fagnano to Taylor's challenge. We mentioned above the arguments between Johann Bernoulli and Taylor. Taylor, although he did not win all the arguments, could certainly dispute with Johann Bernoulli on fairly equal terms. Jones describes these arguments in [1]:-

> Their debates in journals occasionally included rather heated phrases and, at one time, a wager of fifty guineas. When Bernoulli suggested in a private letter that they couch their debate in more gentlemanly terms, Taylor replied that he meant to sound sharp and to "show an indignation".

Jones also explains in [1] that Taylor was a mathematician of far greater depth than many have given him credit for:-

> A study of Brook Taylor's life and work reveals that his contribution to the development of mathematics was substantially greater than the attachment of his name to one theorem would suggest. His work was concise and hard to follow. The surprising number of

major concepts that he touched upon, initially developed, but failed to elaborate further leads one to regret that health, family concerns and sadness, or other unassessable factors, including wealth and parental dominance, restricted the mathematically productive portion of his relatively short life.

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References:

[1] Biography in *Dictionary of Scientific Biography* (New York 1970-1990).

[2] L Conte, G Bernoulli, G C de' Toschi di Fagnano e la sfida di Brook Taylor, *Bul. Inst. Politech. Iacsi* **4** (1949), 36-53.

[3] L Feigenbaum, *Brook Taylor and the method of increments*, Arch. Hist. Exact Sci. 34 (1-2) (1985), 1-140.

[4] A Malet, James Gregorie on tangents and the "Taylor" rule for series expansions, *Arch. Hist. Exact Sci.* **46** (2) (1993), 97-137.

[5] G Maltese, Taylor and John Bernoulli on the vibrating string : aspects of the dynamics of continuous systems at the beginning of the 18th century, *Physis Riv. Internaz. Storia Sci. (N.S.)* **29** (3) (1992), 703-744.

[6] S S Petrova and D A Romanovska, On the history of the discovery of Taylor series (Russian), *Istor.-Mat. Issled. No.* **25** (1980), 10-24; 378.