Math 254: Introduction to Linear Algebra
Lecture Notes #1.2 — Matrices, Vectors, ...

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1. Student Learning Objectives
   - SLOs: Matrices, Vectors, ...

2. Matrices, Vectors; Gauss-Jordan Elimination
   - Matrix – Vector Notation
   - Back to Solving Linear Systems
   - Summarizing

3. Suggested Problems
   - Suggested Problems
   - Lecture–Book Roadmap

4. Supplemental Material
   - Metacognitive Reflection
   - Problem Statements 1.2

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After this lecture you should:

- Know basic language and concepts:
  - Matrices, vectors, and their components
  - Matrix types: square, diagonal, triangular, zero, identity
  - The collection of all \( n \)-vectors, denoted \( \mathbb{R}^n \) is a vector space

- Vector addition

- Know the difference between the Coefficient matrix, and the Augmented matrix; their uses in the solution of linear systems

- Know how to use elimination to identify leading and non-leading (a.k.a. free variables), and when necessary introduce parameters to express all solutions of linear systems.

- Know what **Reduced–Row–Echelon–Form** (RREF) of a Matrix is, and how to achieve it using elementary row operations.
Our current “business” is manipulating linear systems, of the form

\[
\begin{align*}
3x + 21y - 3z &= 0 \\
-6x - 2y - z &= 62 \\
2x - 3y + 8z &= 32
\end{align*}
\]

into a form which reveals the values of \(x\), \(y\), and \(z\):

\[
\begin{align*}
x &= -\frac{3574}{281} \\
y &= \frac{844}{281} \\
z &= \frac{2334}{281}
\end{align*}
\]

We achieve this by cleverly adding/subtracting rows (equations) from each other.
Matrix Notation — “Encoding” the Information

We realize that all the important information is in the coefficients (numbers), and that the variables ($x$, $y$, $z$) just get carried around. We can “encode” all the information about the linear system

\[
\begin{align*}
3x & + 21y - 3z = 0 \\
-6x & - 2y - z = 62 \\
2x & - 3y + 8z = 32
\end{align*}
\]

in a matrix

\[
\begin{bmatrix}
3 & 21 & -3 & 0 \\
-6 & -2 & -1 & 62 \\
2 & -3 & 8 & 32
\end{bmatrix}
\]

or, sometimes:

\[
\begin{bmatrix}
3 & 21 & -3 & 0 \\
-6 & -2 & -1 & 62 \\
2 & -3 & 8 & 32
\end{bmatrix}
\]

Augmented Matrix

Augmented Matrix with Coefficient Matrix and right-hand-side "separated."
Row – Column Indexing

Ponder the matrix “A” with 3 rows, and 4 columns:

\[
A = \begin{bmatrix}
3 & 21 & -3 & 0 \\
-6 & -2 & -1 & 62 \\
2 & -3 & 8 & 32 \\
\end{bmatrix} = \begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
\end{bmatrix}
\]

that is, usually we refer to the entries of a matrix \(A\) (upper-case), using double subscripts \(a_{ij}\) (lower-case); the subscripts \(i\), and \(j\) are “standard” but \(r\) (row) and \(c\) (column) would be more intuitive.

Sometimes you see the notation \(A \in \mathbb{R}^{3 \times 4}\) to denote a 3-by-4 (always [\text{Rows}-by-\text{Columns}]) matrix where the entries are real (\(\mathbb{R}\)) numbers.

\textbf{Note:} The entries can be other mathematical objects, \(e.g.\) complex numbers, \(\mathbb{C}\), polynomials, etc... but we will work with \(\mathbb{R}\) for quite while.
What is a Matrix?

From a computer science point-of-view a matrix can be viewed as *data structure*, and depending on your mood (and/or preference of programming paradigm) you can think of it as e.g. a

- **2-dimensional array**,  
  C/C++ :: double A[3][3]; /* A is a 3-by-3 matrix */  
  C/C++ :: A[0][0] = 1;     /* Assigning 1 to a_{11} */  
  C/C++ :: A[2][2] = 14;    /* Assigning 14 to a_{33} */  
  C/C++ :: Yes, some languages count from 0 to (n-1); others from 1 to n.

- **or a container class**.
Types of Matrices

- When $A \in \mathbb{R}^{n \times n}$, i.e. the matrix has the same number of rows and columns, it is a **square matrix**

- A matrix is **diagonal** if all entries $a_{ij} = 0$ for all $i \neq j$. (Only entries of the type $a_{ii}$ are non-zero.

- A square matrix is **upper triangular** if all entries $a_{ij} = 0$ for all $i > j$. 
Types of Matrices

- A square matrix is **strictly upper triangular** if all entries \( a_{ij} = 0 \) for all \( i \geq j \).

- A square matrix is **lower triangular** if all entries \( a_{ij} = 0 \) for all \( i < j \).

- A square matrix is **strictly lower triangular** if all entries \( a_{ij} = 0 \) for all \( i \leq j \).
Types of Matrices

- A matrix where all entries are zero is (surprisingly?) called a **zero matrix**.

- A square matrix where all diagonal entries are **ones**, and the off-diagonal entries are **zeros**

  \[
  I_n = \begin{bmatrix}
  1 & 0 & \cdots & 0 & 0 \\
  0 & 1 & \cdots & 0 & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  0 & 0 & \cdots & 1 & 0 \\
  0 & 0 & \cdots & 0 & 1 \\
  \end{bmatrix}
  \]

  is called an **identity matrix**. “\(\cdots\)” and “\(\vdots\)” denote padding with 0-entries, and “\(\ddots\)” diagonal 1-entries; filling the matrix out to its full size (whatever that may be).
Matrices of size $n \times 1$ and $1 \times n \Rightarrow \text{"Vectors"}

- A “matrix” with only one column is called a **column vector**:

  $$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

- A “matrix” with only one row is called a **row vector**:

  $$\vec{w}^T = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix}$$

By mathematical convention a **vector** is a **column vector**; so $\vec{v}$ and $\vec{w}$ are (column) vectors. The notation $\vec{w}^T$ is the **transpose** of the vector $\vec{w}$ into a **row vector**.
The entries of a vector are called its **components**; $v_k$ is the $k^{th}$ component of the vector $\vec{v}$.

The set (collection) of all (column) vectors with $n$ components is denoted by $\mathbb{R}^n$; we refer to $\mathbb{R}^n$ as a vector space.

It is easy to visualize vectors in $\mathbb{R}^2$; we can think of the vector $\vec{v}$ as an arrow from the origin $(0, 0)$ to the point $(x, y) = (v_1, v_2)$:

In the figure we have

\[
\begin{align*}
\vec{u} &= \begin{bmatrix} 1 \\ 2 \end{bmatrix}, & \vec{v} &= \begin{bmatrix} 1 \\ 3 \end{bmatrix}, & \vec{w} &= \begin{bmatrix} 2 \\ 2 \end{bmatrix},
\end{align*}
\]

Without confusion, we can just let the terminal points $(x, y) = (v_1, v_2)$ represent the vectors.
Adding Vectors

With:

\[ \vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \]

We can graphically show how to add vectors:

That is

\[ \vec{u} + \vec{v} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \quad \vec{u} + \vec{v} + \vec{w} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}. \]
Making Use of our Matrix – Vector Notation

Now, given a linear system:

\[
\begin{bmatrix}
2x + 8y + 4z &= 2 \\
2x + 5y + z &= 5 \\
4x + 10y - z &= 1
\end{bmatrix}
\]

We can extract the **Coefficient Matrix** (containing the coefficients of the unknown variables in the system)

\[
\begin{bmatrix}
2 & 8 & 4 \\
2 & 5 & 1 \\
4 & 10 & -1
\end{bmatrix},
\]

or the **augmented matrix**

\[
\begin{bmatrix}
2 & 8 & 4 & 2 \\
2 & 5 & 1 & 5 \\
4 & 10 & -1 & 1
\end{bmatrix},
\]

which captures all the information in the linear system.
The Augmented Matrix

Often, we separate the coefficients from the right-hand-side information in the Augmented Matrix:

\[
\begin{bmatrix}
2 & 8 & 4 & 2 \\
2 & 5 & 1 & 5 \\
4 & 10 & -1 & 1 \\
\end{bmatrix}
\sim
\begin{bmatrix}
2 & 8 & 4 & 2 \\
2 & 5 & 1 & 5 \\
4 & 10 & -1 & 1 \\
\end{bmatrix}
\]
Solving Linear Systems Using the Augmented Matrix

We can solve the linear system by manipulating the Augmented Matrix:

\[
\begin{bmatrix}
2 & 8 & 4 & 2 \\
2 & 5 & 1 & 5 \\
4 & 10 & -1 & 1
\end{bmatrix}
/ 2
\]

\[
\begin{bmatrix}
1 & 4 & 2 & 1 \\
2 & 5 & 1 & 5 \\
4 & 10 & -1 & 1
\end{bmatrix}
-2r_1
\]

\[
\begin{bmatrix}
1 & 4 & 2 & 1 \\
0 & -3 & -3 & 3 \\
0 & -6 & -9 & -3
\end{bmatrix}
/ (-3)
\]
Solving Linear Systems Using the Augmented Matrix

\[
\begin{bmatrix}
1 & 4 & 2 & | & 1 \\
0 & 1 & 1 & | & -1 \\
0 & 2 & 3 & | & 1 \\
\end{bmatrix} - 2r_2
\]

\[
\begin{bmatrix}
1 & 4 & 2 & | & 1 \\
0 & 1 & 1 & | & -1 \\
0 & 0 & 1 & | & 3 \\
\end{bmatrix} - r_3
\]

\[
\begin{bmatrix}
1 & 4 & 0 & | & -5 \\
0 & 1 & 0 & | & -4 \\
0 & 0 & 1 & | & 3 \\
\end{bmatrix} - 4r_2
\]

\[
\begin{bmatrix}
1 & 0 & 0 & | & 11 \\
0 & 1 & 0 & | & -4 \\
0 & 0 & 1 & | & 3 \\
\end{bmatrix}, \quad \begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} = \begin{bmatrix}
11 \\
-4 \\
3 \\
\end{bmatrix}
\]
The (Math) World is not limited to 2, or 3 Variables (Dimensions)

We can (easily?) imagine a system of 4 linear equations with 7 unknowns:

$$
\begin{align*}
\begin{vmatrix}
\mathbf{x}_1 & - \mathbf{x}_2 & \mathbf{x}_3 & \mathbf{x}_4 & \mathbf{x}_5 & \mathbf{x}_6 & \mathbf{x}_7 \\
-\mathbf{x}_2 & + 4\mathbf{x}_5 & \mathbf{x}_3 & + 2\mathbf{x}_5 & - 777\mathbf{x}_7 & \mathbf{x}_6 & - 7\mathbf{x}_7 \\
& & & & & 777\mathbf{x}_7 & = 1 \\
& & & & & 777\mathbf{x}_7 & = 2 \\
& & & & & 77\mathbf{x}_7 & = 3 \\
& & & & & 7\mathbf{x}_7 & = 4
\end{vmatrix}
\end{align*}
$$

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Lecture Notes #1.2 — Matrices, Vectors, ... — (18/36)
We can (easily?) imagine a system of 4 linear equations with 7 unknowns:

\[
\begin{aligned}
    x_1 &- x_2 &+ 4x_5 & & & & & & & & & & & & & & & & 7777x_7 &= 1 \\
    & & & + 2x_5 & & & & & & & & & & & & & & & & & & 777x_7 &= 2 \\
    & & & & & & & & & & & & & & & & & & & & & & 77x_7 &= 3 \\
\end{aligned}
\]

so that solving for the leading variables\(^*\) gives:

\[
\begin{aligned}
    x_1 &= 1 + x_2 - 4x_5 - 7777x_7 \\
    x_3 &= 2 - 2x_5 + 777x_7 \\
    x_4 &= 3 - 77x_7 \\
    x_6 &= 4 + 7x_7
\end{aligned}
\]

\(^*\) **Leading variables** are the first ones to appear in each equation (after elimination); here \(x_1, x_3, x_4,\) and \(x_6.\)
Infinitely Many Solutions

If we parameterize (the non-leading, or “free” variables): $x_2 = s$, $x_5 = t$, and $x_7 = u$, we can write the infinitely many solutions:

$$
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  x_4 \\
  x_5 \\
  x_6 \\
  x_7 \\
\end{bmatrix} =
\begin{bmatrix}
  1 & + & s & - & 4t & - & 7777u \\
  2 & s & - & 2t & + & 777u \\
  3 & 0 & - & 77u \\
  4 & t & + & 7u \\
\end{bmatrix}
= \begin{bmatrix}
  1 & 1 & -4 & -7777 \\
  0 & 0 & 0 & 0 \\
  2 & 0 & -2 & 777 \\
  3 & 0 & 1 & 7 \\
\end{bmatrix}
+ \begin{bmatrix}
  0 \\
  1 \\
  0 \\
  0 \\
\end{bmatrix}
+ \begin{bmatrix}
  0 \\
  0 \\
  777 \\
  1 \\
\end{bmatrix}
+ \begin{bmatrix}
  0 \\
  0 \\
  0 \\
  1 \\
\end{bmatrix}
$$

If we plug that into the original system of linear equations we see that it indeed is the (collection of) solution(s)!

Note that $s$, $t$, and $u$ are allowed to take any values in $\mathbb{R}$ (independent of each other)...

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Lecture Notes #1.2 — Matrices, Vectors, ... — (19/36)
What Makes a System “Easy” to Solve?

Three properties make a system “easy” to solve:

**P1** The leading coefficient in each equation is 1.

**P2** The leading variable in each equation does not appear in any other equation.

**P3** The leading variables appear in “natural order,” with increasing indices as we go down the system: $x_1$, $x_3$, $x_4$, and $x_6$ as opposed to any other ordering.

If/when the system does not satisfy these properties, we use elimination to get there...
Another Example...

We go straight to the Augmented Matrix:

\[
\begin{bmatrix}
2 & 4 & -2 & 2 & 4 & 2 \\
1 & 2 & -1 & 2 & 0 & 4 \\
3 & 6 & -2 & 1 & 9 & 1 \\
5 & 10 & -4 & 5 & 9 & 9
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & -1 & 1 & 2 & 1 \\
1 & 2 & -1 & 2 & 0 & 4 \\
3 & 6 & -2 & 1 & 9 & 1 \\
5 & 10 & -4 & 5 & 9 & 9
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & -1 & 1 & 2 & 1 \\
0 & 0 & 0 & 1 & -2 & 3 \\
0 & 0 & 1 & -2 & 3 & -2 \\
0 & 0 & 1 & 0 & -1 & 4
\end{bmatrix}
\]
Another Example...

Keeping P1, P2, P3 in mind...

\[
\begin{bmatrix}
1 & 2 & -1 & 0 & 4 & -2 \\
0 & 0 & 0 & 1 & -2 & 3 \\
0 & 0 & 1 & 0 & -1 & 4 \\
0 & 0 & 1 & 0 & -1 & 4
\end{bmatrix}
\begin{bmatrix}
+ r_3 \\
-r_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & 0 & 0 & 3 & 2 \\
0 & 0 & 1 & -2 & 3 \\
0 & 0 & 1 & 0 & -1 & 4 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\rightsquigarrow r_3 \\
\rightsquigarrow r_2
\end{bmatrix}
\]

leading zeros

\[
\begin{bmatrix}
1 & 2 & 0 & 0 & 3 & 2 \\
1 & 0 & -1 & 4 \\
1 & -2 & 3
\end{bmatrix}
\]

suppressed

for clarity

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... Identifying the Solutions

\[
\begin{bmatrix}
  x_1 & x_2 & x_3 & x_4 & x_5 & \text{value} \\
  1 & 2 & 3 & 2 \\
  1 & -1 & 4 \\
  1 & -2 & 3 \\
\end{bmatrix}
\]

So that:

\[
\begin{aligned}
  x_1 &= 2 - 2x_2 - 3x_5 \\
  x_3 &= 4 + x_5 \\
  x_4 &= 3 + 2x_5 \\
\end{aligned}
\]

\[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  x_4 \\
  x_5 \\
\end{bmatrix} = \begin{bmatrix}
  2 - 2s - 3t \\
  s \\
  4 + t \\
  3 + 2t \\
  t \\
\end{bmatrix} = \begin{bmatrix}
  2 \\
  0 \\
  4 \\
  3 \\
  0 \\
\end{bmatrix} + s \begin{bmatrix}
  -2 \\
  1 \\
  0 \\
  0 \\
  0 \\
\end{bmatrix} + t \begin{bmatrix}
  -3 \\
  0 \\
  1 \\
  2 \\
  1 \\
\end{bmatrix}. 
\]

Parameters: \( x_2 = s, \ x_5 = t \)
Solving a System of Linear Equations

“Elimination Strategy”

We go equation-by-equation from top to bottom ($i$ runs from 1 to $n$):

- For the $i^{th}$ equation: if the leading variable is $x_j$ with non-zero coefficient $c$; divide the equation by $c$ to make the leading coefficient 1.
- Eliminate $x_j$ from all other equations.
- Go to the next equation.

“Exit Strategy”

- If we get $0 = \text{nonzero}$ at any point; then there are no solutions. [STOP]
- If we complete without inconsistencies:
  - **rearrange** the equations so that the leading variables are in “natural order”
  - Solve each equation for the leading variable
  - Choose parameters for the **non-leading variables** [if there are any] (appropriate “alphabet soup”)
  - Express leading variables using parameters.
Reduced Row Echelon Form

After elimination according to this strategy, the matrix is in:

**Reduced Row Echelon Form**

A Matrix is said to be in *Reduced Row Echelon Form* if it satisfies the following conditions:

1. If a row has non-zero entries, then the first non-zero entry is a 1, called *the leading 1* (or *pivot*) of this row.
2. If a column contains a leading 1, then all other entries in that column are 0. [Elimination is Complete]
3. If a row contains a leading 1, then each row above it contains a leading 1 further to the left. [Sorting of Rows]

The last condition implies that rows of 0’s, if any, must appear at the bottom of the matrix.
We get to *Reduced Row Echelon Form* by performing

**Elementary Row Operations**

- Divide a row by a non-zero scalar
- Subtract a multiple of a row from another row
- Swap two rows

This strategy of solving linear systems by reduction to Reduced Row Echelon Form is referred to as *Gaussian Elimination*, or *Gauss-Jordan Elimination*.

Gauss (1777–1855), Jordan (1842–1899); but the Chinese used it loooong before that.

“Gauss-Jordan Elimination” \(\rightsquigarrow\) RREF
“Gaussian Elimination” \(\rightsquigarrow\) REF (leading variables NOT 1’s; *LU*-factorization)
Available on “Learning Glass” videos:

(1.2.1) Find all solutions to a 2-by-3 linear system using elimination.

(1.2.3) Find all solutions to a 1-by-3 linear system using elimination.

(1.2.9) Find all solutions to a 3-by-6 linear system using elimination.

(1.2.11) Find all solutions to a 4-by-4 linear system using elimination.

(1.2.18) Determine which matrices are in RREF.

(1.2.21) Find values of matrix entries so that the resulting matrix is in RREF.
# Lecture – Book Roadmap

<table>
<thead>
<tr>
<th>Lecture</th>
<th>Book, [GS5–]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>§2.2</td>
</tr>
<tr>
<td>1.2</td>
<td>§1.1, §1.3, §2.1, §2.3</td>
</tr>
</tbody>
</table>
Metacognitive Exercise — Thinking About Thinking & Learning

<table>
<thead>
<tr>
<th>I know / learned</th>
<th>Almost there</th>
<th>Huh?!?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right After Lecture</td>
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<tr>
<td>After Thinking / Office Hours / SI-session</td>
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<td></td>
</tr>
<tr>
<td>After Reviewing for Midterm/Final</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(1.2.1) Find all solutions to the linear system using elimination:

\[
\begin{align*}
 x + y - 2z &= 5 \\
 2x + 3y + 4z &= 2
\end{align*}
\]

(1.2.3) Find all solutions to the linear system using elimination:

\[
\begin{align*}
 x + 2y + 3z &= 4
\end{align*}
\]
(1.2.9), (1.2.11)

(1.2.9) Find all solutions to the linear system using elimination:

\[
\begin{align*}
    x_1 &+ 2x_2 & & & x_4 &+ 2x_5 &- x_6 &= 2 \\
    x_1 &+ 2x_2 & & & + x_5 &- x_6 &= 0 \\
    x_1 &+ 2x_2 &+ 2x_3 & & - x_5 &+ x_6 &= 2
\end{align*}
\]

(1.2.11) Find all solutions to the linear system using elimination:

\[
\begin{align*}
    x_1 & & & + 2x_3 &+ 4x_4 &= -8 \\
    x_2 &- 3x_3 &- x_4 &= 6 \\
    3x_1 &+ 4x_2 &- 6x_3 &+ 8x_4 &= 0 \\
    -x_2 &+ 3x_3 &+ 4x_4 &= -12
\end{align*}
\]
(1.2.18) Determine which of the matrices are in Reduced Row Echelon Form:

a. \[
\begin{bmatrix}
1 & 2 & 0 & 2 & 0 \\
0 & 0 & 1 & 3 & 0 \\
0 & 0 & 1 & 4 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

b. \[
\begin{bmatrix}
0 & 1 & 2 & 0 & 3 \\
0 & 0 & 0 & 1 & 4 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

c. \[
\begin{bmatrix}
1 & 2 & 0 & 3 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 2
\end{bmatrix}
\]

d. \[
\begin{bmatrix}
0 & 1 & 2 & 3 & 4
\end{bmatrix}
\]
(1.2.21) For which values of $a$, $b$, $c$, $d$, and $e$ is the following matrix in reduced-row-echelon-form?

\[
\begin{bmatrix}
1 & a & b & 3 & 0 & -2 \\
0 & 0 & c & 1 & d & 3 \\
0 & e & 0 & 0 & 1 & 1
\end{bmatrix}
\]
Two Separate Websites

- **iClicker Reef**
  - [https://app.reef-education.com/](https://app.reef-education.com/)
  - **Reef-Account**
    - Tied to iClicker or/and Reef-app
    - Records scores

- **Blackboard at San Diego State University**
  - [https://blackboard.sdsu.edu/](https://blackboard.sdsu.edu/)
  - **GRADEBOOK**
Goal: Your Points in the GRADEBOOK!

How do we get there?

1. Set up a **Reef-Account**
   - Create and log into your reef-account [FREE]
     - iClicker Remote ➔ FREE account
     - iClicker Reef app:
       - $14.99 / 6 months; $23.99 / 1 year;
       - $32.99 / 2 years; $47.00 / 4 years.
   - Add your remote / app to your account
   - Gory details:
   - Once you can see your scores in the Reef “gradebook,” you can move to step 2.
Goal: Your Points in the GRADEBOOK!
How do we get there?

2. Link your Reef and Blackboard accounts
   - Log into Blackboard.
   - Go to [Math 254]
   - Click [Clicker cloud SETUP] in the left menu.
   - Sign into your Reef account from the window that opens. (Allow the popup from Blackboard)
   - Once signed in, click through to Math 254 on the reef site.

3. Email me — I’ll trigger a re-synchronization of the Reef and Blackboard gradebooks, and your scores will show up in Blackboard.