Numerical Matrix Analysis
Lecture Notes #1 — Introduction

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Outline

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MSc. Engineering Physics, Royal Institute of Technology (KTH), Stockholm, Sweden. Thesis Advisers: Michael Benedicks, Department of Mathematics KTH, and Erik Aurell, Stockholm University, Department of Mathematics. Thesis Topic: “A Renormalization Technique for Families with Flat Maxima.”

Figure: Bifurcation diagram for the family $f_{a, \frac{1}{2}}$ [BLOMGREN-1994]

Figure: The noisy (SNR = 4.62 dB), and recovered space curves. Notice how the edges are recovered. [Blomgren-1998]
- Research Associate, Stanford University, Department of Mathematics. Main Focus: Time Reversal and Imaging in Random Media (with George Papanicolaou, et al.)

**Figure:** Comparison of the theoretical formula for a medium with $L = 600 \, m$, $a_e = 195 \, m$, $\gamma = 2.12 \times 10^{-5} \, m^{-1}$. [LEFT] shows a homogeneous medium, $\gamma = 0$, with $a = 40 \, m$ TRM (in red / wide Fresnel zone), and a random medium with $\gamma = 2.12 \times 10^{-5}$ (in blue). [RIGHT] shows $\gamma = 0$, with $a = a_e = 195 \, m$ (in red), and $\gamma = 2.12 \times 10^{-5}$, with $a = 40 \, m$ (in blue). The match confirms the validity of [the theory]. [Blomgren-Papanicolaou-Zhao-2002]
Professor, San Diego State University, Department of Mathematics and Statistics. Projects: Computational Combustion, Biomedical Imaging (Mitochondrial Structures, Heartcell Contractility, Skin/Prostate Cancer Classification).

Figure: [Left] Phase-space projections produced by the time coefficients of the POD decomposition of the rotating pattern shown in [Right]. [Blomgren-Gasner-Palacios-2005]
## Contact Information

<table>
<thead>
<tr>
<th>Office</th>
<th>GMCS-587</th>
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<tbody>
<tr>
<td>Email</td>
<td><a href="mailto:blomgren.peter@gmail.com">blomgren.peter@gmail.com</a></td>
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<tr>
<td>Web</td>
<td><a href="http://terminus.sdsu.edu/SDSU/Math543/">http://terminus.sdsu.edu/SDSU/Math543/</a></td>
</tr>
<tr>
<td>Phone</td>
<td>N/A</td>
</tr>
<tr>
<td>Office Hours</td>
<td>TuTh: 2:00pm – 3:15pm, W 1:30pm – 2:45pm and by appointment</td>
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Fun Times... ⇒ Endurance Sports

- **Triathlons:**
  - (12) Ironman distance $(2.4 + 112 + 26.2)$ — $11:48:57$ [PR]
  - (16) Half Ironman distance — $5:14:20$

- **Running**
  - (1) Trail Double-marathon (52 miles) — $10:59:00$
  - (4) Trail 50-mile races — $9:08:46$
  - (6) Trail 50k (31 mile) races — $5:20:57$
  - (13) Road Marathons — $3:26:19$ (7:52/mi)
  - (23) Road/Trail Half Marathons — $1:36:25$ (7:21/mi)
Math 543: Literature

“Required” —


“Required” — (Supplemental)

Class notes and class web-page.

“Optional” — (Classic, Comprehensive Reference)

33\frac{1}{3}\% \text{ Homework: both theoretical, and implementation (programming) — C/C++ or Fortran are the recommended languages, but feel free to program in 6510 assembler, Java, M$-D^b$, Python, Haskell, or Matlab...}

33\frac{1}{3}\% \text{ Midterm: [Take-Home].}

33\frac{1}{3}\% \text{ Final Project: [Complete details TBA].}
Expectations and Procedures, I

- Most class attendance is “OPTIONAL” — Homework and announcements will be posted on the class web page. If/when you attend class:
  - Please be on time.
  - Please pay attention.
  - Please turn off mobile phones.
  - Please be courteous to other students and the instructor.
  - Abide by university statutes, and all applicable local, state, and federal laws.
Expectations and Procedures, II

- Please, turn in assignments on time. (The instructor reserves the right not to accept late assignments.)
- The instructor will make special arrangements for students with documented learning disabilities and will try to make accommodations for other unforeseen circumstances, e.g. illness, personal/family crises, etc. in a way that is fair to all students enrolled in the class. **Please contact the instructor EARLY regarding special circumstances.**
- Students are expected **and encouraged** to ask questions in class!
- Students are expected **and encouraged** to make use of office hours! If you cannot make it to the scheduled office hours: contact the instructor to schedule an appointment!
Expectations and Procedures, III

- Missed midterm exams: Don’t miss exams! The instructor reserves the right to schedule make-up exams, make such exams oral presentation, and/or base the grade solely on other work (including the final exam).

- Missed final exam: Don’t miss the final! Contact the instructor ASAP or a grade of incomplete or F will be assigned.

- Academic honesty: submit your own work — but feel free to discuss homework with other students in the class!
The following Honesty Pledge must be included in all programs you submit (as part of homework and/or projects):

I, (your name), pledge that this program is completely my own work, and that I did not take, borrow or steal code from any other person, and that I did not allow any other person to use, have, borrow or steal portions of my code. I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies.

Work missing the honesty pledge may not be graded!
Honesty Pledges, II

- Larger reports must contain the following text:
  
  - I, (your name), pledge that this report is completely my own work, and that I did not take, borrow or steal any portions from any other person. Any and all references I used are clearly cited in the text. I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies. Your signature.

- Work missing the honesty pledge may not be graded!
Math 543: Computer Resources

You need access to a computing environment in which to write your code; — you may want to use a combination of Matlab (for quick prototyping and short homework assignments) and C/C++ or Fortran (or something completely different, like Java or M$-D^b$).

Class accounts for the computer lab(s) will be available.

You can also use the Rohan Sun Enterprise system or another capable system. [http://www-rohan.sdsu.edu/raccts.html]

Free C/C++ (gcc) and Fortran (f77) compilers are available for Linux/UNIX.

You may also want to consider buying the student version of Matlab: http://www.mathworks.com/

SDSU students can download a copy of matlab from http://www-rohan.sdsu.edu/~download/matlab.html

[Licensing Subject to Change With Minimal Notice]
Math 543: Introduction — What you should know already

**Prerequisite:** Math 541

541 ⇒ *Intro to Numerical Analysis and Computing*
- Solution of equations of one variable, direct methods in numerical linear algebra, least squares approximation, interpolation and uniform approximation, quadrature.

**Pre-Prerequisite:** Math 254 or Math 342A

254 ∩ 342A ⇒ *Basic Linear Algebra*
- Vectors, Matrices, Eigenvalues and Eigenvectors.

\[ \bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = \begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \\ \cdots \\ \bar{a}_n \end{bmatrix} \]
Math 543: Introduction — Why???

Solution of linear systems and eigenvalue problems show up in many applications in applied & computational mathematics / sciences / engineering.

Although we probably know about Gaussian Elimination for solving for

\[ \bar{x} = A^{-1}\bar{b} \]

in infinite precision, finding this solution (or at least a good approximation thereof) in finite precision (i.e. on a computer) is sometimes a challenge — especially if we need the solution fast.
The computational complexity (number of operations needed) for Gaussian Elimination is $O(n^3)$, which is quite slow as $n$ grows “large.”

Applications (sources of Numerical Linear Algebra problems):
- Solution of partial differential equations (PDEs)
- Optimization (Operations Research)
- Model Analysis and Fitting (Least Squares)
- Image Processing
- Protein Folding
- DNA sequencing, etc. etc. etc.
Math 543: Introduction — What we will learn

\[ A\tilde{x} = \tilde{b}, \quad A\tilde{x} = \lambda\tilde{x}, \quad Q^T AQ = \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n) \]

- QR-Factorization / Least Squares
- The SVD
- Conditioning and Stability
- Gaussian Elimination, Pivoting
  \[ \Rightarrow \text{LU- and Cholesky-factorization} \]
- Eigenvalue Problems
- Iterative Methods
  \[ \Rightarrow \text{Arnoldi, GMRES, Lanczos, Conjugate Gradients} \]